## PROBLEMS FOR COMPETITIONS AND OLYMPIADS

### Junior Level

**C.O:5219.** Find positive real numbers $x$ and $y$ such that $2x[y] = [x]+y$ and $2y[x] = x + [y]$.

*Alexandru Blaga*, Satu Mare

**C.O:5220.** Prove that $\frac{1}{1^2} + \frac{1}{2^2} + \ldots + \frac{1}{2011}^{2} \leq 2 - \frac{1}{2011}$.

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**C.O:5221.** Find all real values of $a$ for which the set $[1, a] \cap [2, 3]$ is an interval.

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**C.O:5222.** The middle segments of an isosceles triangle have the lengths equal to 3 and 7. Find the perimeter of the triangle.

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**C.O:5223.** Show that:

$$\frac{x^2}{(x+2y)(x+2z)} + \frac{y^2}{(y+2z)(y+2x)} + \frac{z^2}{(z+2x)(z+2y)} \geq \frac{1}{3},$$

for any $x, y, z > 0$.

*Petre Bătrânețu*, Galați

**C.O:5224.** Let $ABCD$ be a regular tetrahedron of side lengths 1. Points $M$, $N$, $P$, $Q$ lies on the sides of the tetrahedron such that 5 sides of tetrahedron $MNPQ$ have the lengths $\frac{1}{2}$. Find the volume of the tetrahedron $MNPQ$.

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**C.O:5225.** Solve in positive integers the equation $\frac{n^6 - 6^n}{7} = m^2$.

*Ionel Tudor*, Călugăreni, Giurgiu

**C.O:5226.** Let $a > 0$ be a real number. Find all real values of $x$ such that $x$, $a + x$, $2a + x$ are the sidelengths of an acute triangle.

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### Senior Level

**C.O:5227.** Show that the integer:

$$A = \left[\frac{2011}{1}\right] + \left[\frac{2011}{2}\right] + \ldots + \left[\frac{2011}{2011}\right]$$

is even.

*Adrian Zahariuc*, U.S.A.

**C.O:5228.** Let $n$ be a positive integer. Show that $2^n + 1$ does not have prime divisors of the form $8k + 7$.

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**C.O:5229.** Let $P$ be a point inside the regular tetrahedron $ABCD$ of side lengths 1. Prove that $PA + PB + PC + PD < 3$.

Polish Olympiad

**C.O:5230.** Let $a$, $b$, $c$ be positive real numbers with $a^2 + b^2 + c^2 = 3$. Show that $a + b + c \geq a^2b^2 + b^2c^2 + c^2a^2$.

*Valeriu Răchită*, Mangalia

**C.O:5231.** Let $f : [a, b] \to \mathbb{R}$ be a continuous function such that for any $x \in [a, b)$ there exists $y \in (x, b]$ such that $f(x) \leq f(y)$. Prove that $f(x) \leq f(b)$, for all $x \in [a, b]$.

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**C.O:5232.** Show that for any matrix $A \in \mathcal{M}_2(\mathbb{R})$ there exist the matrices $X, Y \in \mathcal{M}_2(\mathbb{R})$ such that $A = X^3 + Y^3$ and $XY = YX$.

*Vlad Matei*, Bucharest

**C.O:5233.** Find all integers $n \geq 3$ for which there exists a convex polygon $A_1A_2\ldots A_n$ such that:

$$\sum \frac{\sin A_1}{\sin A_2 \cdot \sin A_3 \cdot \ldots \cdot \sin A_n + n - 1} = 1.$$

*Dan Ştefan Marinescu*, Hunedoara

**C.O:5234.** Let $n$ be a positive integers and let $f : [0, 1] \to \mathbb{R}$ be an increasing function. Show that:

$$\int_0^{\frac{1}{n}} f(x)\mathrm{d}x \leq \int_0^1 x^{n-1} f(x)\mathrm{d}x \leq \int_{1-\frac{1}{n}}^1 f(x)\mathrm{d}x.$$

*Călin Popescu*, Bucharest

## RUBRICA REZOLVITORILOR DE PROBLEME

**Până la 30 iulie 2011, au trimis soluţii la problemele propuse următorii elevi:**

**ALBA IULIA (ALBA)** *C. Th. „Alexandru Domşa“* cl.IX Munteanu Răzvan (80); *C. Th. „Apulum“* cl.VI Grozav Robert (60), cl.VII Vinţan Mădălina (50+90).
**ALBEŞTI DE MUŞCEL (ARGEŞ)** *fără menţiune de şcoală*: cl.X Manta Maria (30).
**ALEŞD (BIHOR)** *Ş.g. „Constantin Şerban“* cl.VII Gui Andreea (80).
**ARAD (ARAD)** *Lic. „Adam Müller Guttenbrunn“* cl.V Bodescu Adrian (280), Pavel Cristian (90), cl.VI Mariş Mihai Bogdan (90+160), cl.VII Baciu Alexandra (100), Banciu Alexandra (100), Donjan David (100+100), Fizedean Mihai (90+100), Hanc Ioachim (110), Ignat Patrik (100), Iordan Andrada (100), Lata Teofil (100), Mănăilă Roberto (100), Mănăilă Roberto Ronaldo (110), Roman Bogdan (100+100), Szikler Patrik (100), cl.VIII Iancu Denisa (100+90); *C.N. „Vasile Goldiş“* cl.VIII Bocăniciu Camil (100+140).