PROBLEMS FOR COMPETITIONS AND OLYMPIADS

Junior Level

C.O:5211. Let x, y, z be real numbers with $x+y+z=1$. Prove that:

$$x^3+y^3+z^3 \geq 3xyz+3(x-y)(z-y).$$

Emil C. Popa, Sibiu

C.O:5212. Consider a triangle ABC and let B', C' be the midpoints of the sides AC, AB respectively. Show that if $AC+2\cdot BB' = AB+2\cdot CC'$, then triangle ABC is isosceles.

Dumitru Barac, Sibiu

C.O:5213. Find all positive integers n such that

$$1!+4!+7!+\ldots+(3n+1)!$$

is a perfect square, where $x! = 1\cdot 2\cdot 3\cdot \ldots \cdot x$, $x\in\mathbb{N}$.

Mugur Acu, Sibiu

C.O:5214. Prove that there exists a multiple of 2011 having only the digit 3.

Dumitru Acu, Sibiu

Senior Level

C.O:5215. Show that there exists no prime p such that $3^p+19(p-1)$ is a perfect square.

Dumitru Acu, Sibiu

C.O:5216. Find all integers n, $n\geq 2$, such that

$$\operatorname{tr}^2(A^*)-\operatorname{tr}({A^*}^2) = \left[\operatorname{tr}^2(A)-\operatorname{tr}(A^2)\right]\cdot\det(A), \quad (\forall)A\in\mathcal{M}_n(\mathbb{C}),$$

where A^* is the adjoint matrix of A, and $\operatorname{tr}(A)$ is the trace of A.

Dumitru Barac, Sibiu

C.O:5217. Prove that

$$4 \leq |1-4z|+|4z^2-z+4| \leq 14,$$

for any $z\in\mathbb{C}$ with $|z|=1$.

Doriana Dorca, Sibiu

C.O:5218. Find the limit

$$L=\lim_{x\to\infty}\left(x^4\ln^2\frac{x+1}{x}-x^2\mathrm{e}^{-\frac{1}{x}}\right).$$

Livia Băcilă, Sibiu