PROBLEMS FOR COMPETITIONS AND OLYMPIADS

Junior Level

C.O:5075. Let x, y be non-zero integers such that $9x^2 + 6xy - 6y^2 = = x - y$. Prove that $x - y$ is a square and is greater than of equal to 9.

Gheorghe Iurea, Iaşi

C.O:5076. Let ABC be a triangle and let A', B', C' be the midpoints of the sides BC, CA, AB. A point P lies on AA'. Prove that the parallel lines from B' and C' to BP and CP intersect on AA'.

Temistocle Bîrsan, Iaşi

C.O:5077. Consider a regular polygon $A_1A_2A_3...A_{2010}$. Find the number of trapezoids $A_iA_jA_kA_l$ having all vertices among the vertices of the polygon.

Gabriel Popa and *Paul Georgescu*, Iaşi

C.O:5078. Solve in integers the equation $6\left(a^2 - ab + b^2\right) = 31(a + b)$.

Mihai Haivas, Iaşi

Senior Level

C.O:5079. Find the first digit after the decimal point of the number

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{2n}, n \in \mathbb{N}^*.$$

Valentina Blendea and *Gheorghe Blendea*, Iaşi

C.O:5080. Let x, y, z be positive real numbers. Prove that:

$$\frac{x}{y^2 + yz + z^2} + \frac{y}{z^2 + zx + x^2} + \frac{z}{x^2 + xy + y^2} \geq \frac{3}{x + y + z}.$$

Marius Pachiţariu, student, Princeton, U.S.A.

C.O:5081. Describe all quadrilaterals $ABCD$ for which there exists a point M in the same plane such that every line passing through M divides the quadrilateral into two polygons with the same perimeter.

Gheorghe Iurea, Iaşi

C.O:5082. Let $p = 2^n + 1$, $n > 2$, be a prime. Show that p divides $5^{2^{n-1}} + 1$.

Adrian Zanoschi, Iaşi