PROBLEMS FOR COMPETITIONS AND OLYMPIADS

Junior Level

C.O:5059. Find all rational numbers a such that $\dfrac{2a^2 - 5a + 7}{2a - 5}$ is an integer.

Vasile Chiriac, Bacău

C.O:5060. Show that the number $1 + 3 + 3^2 + \ldots + 3^{2009}$ is divisible by 4, but not by 8.

Ovidiu Pop, Satu Mare

C.O:5061. Each unit square of a 2009×2009 table is occupied by a checker. Every checker is moved into another unit square, sharing a common side with the square initially occupied. Show that at least one unit square is left empty.

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C.O:5062. Two altitudes of a triangle have the lenghts 2 and 3 respectively. Show that the third altitude is less than 6.

Lucian Braia, Bacău

Senior Level

C.O:5063. Consider an integer $n > 0$ and $a = 7 + 7^2 + \ldots + 7^n$. Show that $7!$ divides a if and only if 36 divides n.

Ovidiu Pop, Satu Mare

C.O:5064. In a triangle ABC one has $2 \sin A + \sin B \sin C = \dfrac{1+\sqrt{17}}{2}$. Show that the triangle is isosceles.

Adrian Stan, Buzău

C.O:5065. Let ABC be a triangle and let I be the incenter. Line AI meets BC at D and the circumcircle at E. Show that $AI = IE$ if and only if $ID = DE$.

Dinu Şerbănescu, Bucharest

C.O:5066. Show that in any subset of $\{2, 3, \ldots, 2009\}$ containing 15 relatively coprime numbers, at least one is a prime number.

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