Ghidul profesorului

An Unlikely Bestseller: *The Charm of Old-Fashioned Geometry*

by Bogdan D. Suceavă

În 1983, volumul lui Viorel Gh. Vodă, *Vraja geometriei demodate*, apărut la Editura Albatros, a produs un succes de public surprinzător pentru o carte de geometrie euclidiană avansată. Arta narativă a autorului, de o neîndoielnică valoare, îşi păstrează prospeţimea şi peste decenii, iar stilul confesiv, purtător al unei viziuni subiective, împărtăşeşte până azi credibil pasiunea autorului pentru matematică. Privită după atâtea decenii, performanţa editorială a acestui volum rămâne remarcabilă şi ne invită să reflectăm asupra unui set de calităţi editoriale: cum de a fost posibil acel justificat succes, tradus într-un tiraj de zeci de mii de exemplare? Oare poate fi un asemenea succes de audienţă replicat în zilele noastre? La o atentă analiză vom descoperi că, la începutul anilor optzeci, exista în România un public care putea aprecia literatură de popularizare ştiinţifică şi că un asemenea volum răspundea unei nevoi din epocă de a prezenta şi de a asimila informaţia. Marele secret al autorului pare să fi fost identificarea unui filon de discurs accesibil, în bună rezonanţă cu spiritul vremii sale. Mai mult de atât, în anii optzeci întregul discurs al geometriei euclidiene avansate se regăsea într-un final de paradigmă şi unele lucrări scrise de autori din diverse culturi matematice reflectau asupra felului cum geometria ar trebui predată sau prezentată (unul dintre aceştia a fost Philip J. Davis, cu articolul indicat la bibliografie). Viitoarele succese editoriale ale pieţei de carte din România vor trebui să ţină seama de cum se schimbă în timp atât interesul audienţelor, cât şi stilul de prezentare, pentru a răspunde cât mai bine aşteptărilor celor interesaţi de matematică.

We examine the intellectual and editorial conditions of a particularly successful book written for large audiences by Viorel Gh. Vodă in 1983. Our motivation to present this casebook lies in the question: Are similar stylistic options possible today in the present editorial environment? How technical could an author afford to be when addressing a wide mathematical audience? When is it possible to do so?

An Editorial Surprise. In the fall of 1983, the Romanian bookstores distributed a 290-pages small format paperback produced by Albatros Press, titled *The Charm of the Old-Fashioned Geometry* (in original: *Vraja geometriei demodate*) [28]. It sold rather quickly an extensive printing, although it was not in any ways related to any school curriculum or directly related to the content of the local exams, on a market where problem solving supplements usually took the lion's share. It sold better than some crime novels [20]. It did not come from one of the traditional publishers of STEM books, e.g. *Editura Tehnică* or *Editura Ştiinţifică şi Enciclopedică*. The author of this volume on

geometry was not a geometer, but a statistician, who definitely knew very well the craft. The topic was proudly presented from the title as old-fashioned, and usually this is an editorial turn-off, but in this case, being about mathematics, somehow it works. In the end of the day, several cohorts of high-school students read this book. This happened in a decade when the Romanian team ranked first in the International Mathematical Olympiads, and a very stimulating intellectual atmosphere was permeating a younger generation of scholars.

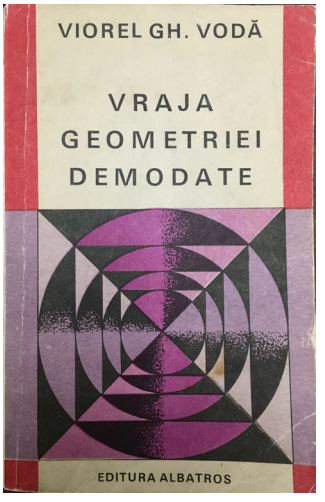

Fig. 1. The cover of *The Charm of the Old-Fashioned Geometry*

How was such an audience success possible? Is this possible to attract genuine interest with a mathematical text? What is the secret of writing an engaging mathematical book that truly reaches large audiences and naturally creates genuine impact? There are many factors involved in the success of this particular case, and perhaps there is an important takeaway in this story.

The Style. It is quite possible that much of the *Charm*'s success lies in the confessional style, the coming-of-age story blended in with good personal anecdotes. One can divagate in a mathematical text, but only with the right measure. The author pursues a consistent narrative flow, while the anecdotes do not weaken the main storyline. There is always something interesting and catchy in the stories starting up with the teenage years, rooted in the pure ideas of a universe when all us hope and learn. Vodă's narrative angle is: this is what I found truly fascinating in the years I grew up, let me share it with you.

E.g., in section 1.5 Vodă's jokes focus on the mathematician's loneliness when he writes:

> ,,In my family, e.g. nobody reads me. Two of my family members have mitigating circumstances, they are still almost illiterate, while the others are telling me they would rather prefer Josef Toman or Raymond Chandler, and this is the mildest insult I hear.

> Let my introduce you now a little bit to my working workshop. First of all, please do know that being a writer of popular science, as I dare to consider myself, is not at all an easy task. Well, one may say, but while you complain it's difficult, who is asking you to write, nobody is forcing you to do it! That might be, you know, but writing is like love, a difficult burden..."

That the mathematician is most of the times alone in her/his passion is well-known, but how to talk about this to young enthusiasts? The idea is discussed extensively also in the documentary [8].

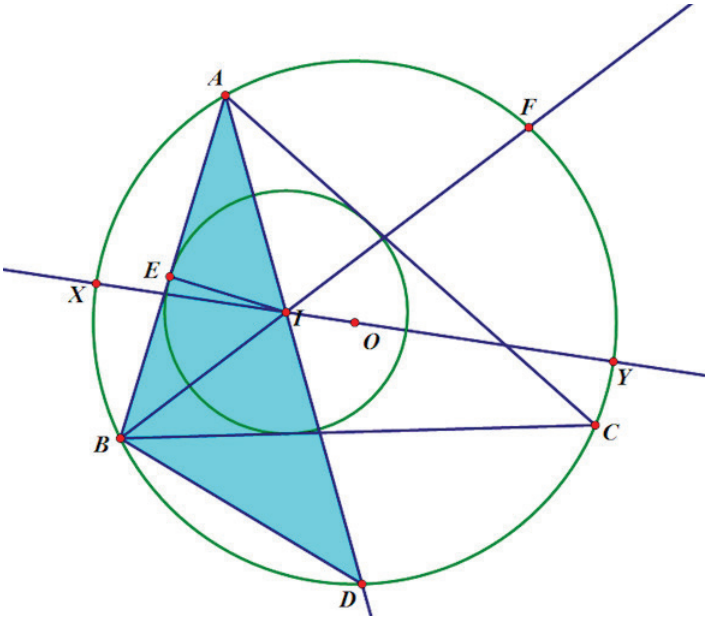

Fig. 2. Calculating the distance between the circumcenter O and the incenter I involves, by a synthetic approach, the use of the power of the point with respect to the circle and an application of the Law of Sines. Can such material be presented to a non-specialised audience? How could such a presentation be motivated from a literary standpoint?

The Historical Context. When Viorel Gh. Vodă's book was released, the author was perfectly aware that his potential audience was the large school-age academic population existent at that time. There may have been readers who could pick up this book just for fun, but there were young people who wanted to be informed and entertained in the same time. The academic experience of a teenager growing up in those years was full of turning-point life-changing national examinations, which were perceived by quite a few as a borderline traumatic experience. Filtering one's skill to support an emerging economy's interest in producing skilled engineers was not a process without secondary consequences.

There was a high-school admission exam which required mathematics, followed by yet another test, after the 10th grade and before the 11-grade, which for many academic specialities included and required mathematics. There was also a high-school graduation exam, with mandatory mathematics including both linear algebra and basic calculus, and, finally, there was an admission into college exam that for all STEM specialities required mathematics. This was the academic environment in which the monthly *Gazeta matematică* used to sell over 100,000 copies [20], quite a large number out of them by subscriptions. There was an army of very young problem solvers taught that mathematics is fun since middle school, and the principal promoters of this vision were the middle-school and high-school teachers.

Geometry approached by the synthetic method was introduced in the 6th grade, and an axiomatic approach was used in the 9th grade. The presentation was adapted after the North American model (quite surprisingly for Eastern Europe at that time), based on a local textbook inspired from the North American School Mathematics Study Group's efforts, citing in the introduction E.E. Moise's classical text [15]. At some point, the Nine Point Circle theorem (Theorem IX in [3], present in the 7th grade textbook, as well in the widely used [21], see Figure 3; see also [12]), was asked as a problem in the admission to high-school national examination. In this context, Viorel Gh. Vodă knew that he might stand a chance to find an interested audience for a tongue-in-cheek with serious content geometry book.

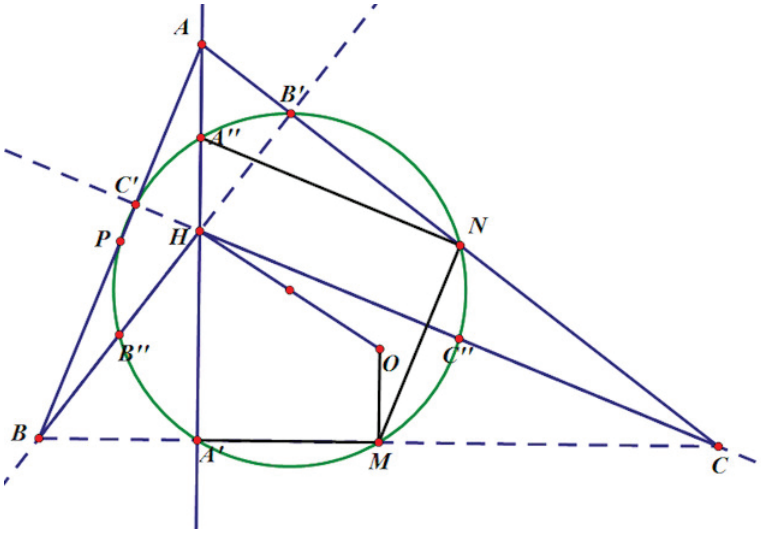

Fig. 3. Brianchon and Poncelet proved in 1821 the Nine-Point circle theorem, stating that in any triangle the midpoints of sides, the feet of the altitudes, and the midpoints of segments joining the vertices with the intersection of altitudes lie all on the same circle.

Nevertheless, the local audiences were quite familiar with the very idea of presenting mathematics to non-specialists. This was the outcome of several decades of skilled efforts from excellent authors, mainly Florica T. Câmpan (1906–1993), who taught history of mathematics at the university in Iaşi, and had a long editorial collaboration with the same Albatros Press that published later Vodă, and George Ştefan Andonie (1911–1998), author of a comprehensive three volume *History of Mathematics in Romania* [1]. The bookstore displays have seen such books before.

Furthermore, Vodă was not at his first attempt to meet the expectations of wide audiences, as he published before [23] and [26], both ushering the pathway for his best work.

The Challenge. The Charm of Old-Fashioned Geometry starts with a longer multi-layered foreword, to create a pleasant narrative environment. At pp. 15-16, the author includes a subsection titled *What I actually wish to do*, where he writes plainly "I am trying to advertise for geometry (for the old-fashioned kind, of course). I admit I am not the most qualified for the matter, but the enchanted amateurs generally manage to present a better image to a certain mathematical area than some very qualified specialists". It's a catchy angle, the reader's curiosity is picked up, the author shows himself as a humble enthusiast and summons no authority on his behalf.

Then, Vodă writes that he wishes to put together some sort of "course" of "synthetic geometry", "spiced up with the smiles needed to keep the reader awake." He is aware that mathematics might very well repel, and he is addressing this matter with untamed humor. As for his inspiration point in writing this very book, he writes:

> ,,The actual work [on this book] started in a tropical July evening in 1980, rather merciless for those still swarming at that date throughout Bucharest. I was laying in gaffer Aurică's armchair, in his rather unfortunately noisy garden [...]"

This is the literary trick. Just imagine that mathematics is not a burden needed to pass some gruesome national examinations. Just imagine it's a leisure activity, something perfectly suitable for a relaxing summer evening. This might work in any culture, at any time, perhaps with different mathematical content.

Vodă reflects upon the Golden Era of triangle geometry, when F.-G.M.'s monographs were written, with focus on the geometry volume in that series. In the next few pages, he writes a brief outline of the history of mathematics, concluding (p. 21) that

> ,,It's rather difficult to gather today what are the actual trends in the area of synthetic geometry. Its charm still relies perhaps in the ingenuity of disparate properties."

If there is a charm to that, then there is a well-defined tension between this thought and the hope for a method to teach, learn, and ultimately do

mathematics. Viorel Gh. Vodă is a trained mathematician who argues for the beauty of original intricate arguments and he's willing to advocate with passion that such a beauty exists. Vodă describes it as a *charm* more suitable for a hot summer evening rather than any day of industrious endeavor. Viewed as such, mathematics is presented as a hedonistic activity, closer to contemplation and leisure, rather than being associated to any sort of tedious effort.

This was the discourse that appealed to a good segment of audience in that time. This intelligent approach is quite similar to Yakov Perelman's idea from [16], a volume published first in 1913, which turned out into a bestseller in the USSR and other countries. The author presented successfully fundamental scientific ideas in [17, 18], and Vodă's approach is closer to these writing strategies, except that he can afford to go more technical, quite technical, because he knew the interested audience existed in his particular cultural context.

The Content. The first chapter is titled *Dilemma (Partially Solved)*, which starts with the properties of the orthic triangle. The text includes figures, and this helps the reader to follow along. The starting point is that the sides of the orthic triangle are antiparallel to the sides of the initial triangle. Then it is proved that the reflections of the orthocenter in the sides of the triangle lie on the circumcircle. Then the story focuses on several distances between the vertices of the triangle and the orthocenter, needed anyways later in forthcoming proofs. The author points out that the area of a triangle in the Euclidean plane equals the product between the semiperimeter of the orthic triangle and the circumradius of the initial triangle, $\mathcal{A} = s_0 R$, appears first in [21]. Once these proofs are detailed, the spirit of the memoir takes over, and the author shows how certain problems were included in the textbook used in his time, at the end of the 1950s. He's describing how his high-school teacher, Nicolae Manolescu, was a co-author, together with a medical doctor, of a monograph in biomathematics, where mathematical methods were used to investigate kidney maladies.

This is just a sample of an unexpected narrative turning point, a mark of Vodă's style. If we chat about mathematics like it's a hot summer evening, why can't we go on a tangent and remember stories from a few decades ago? This is something that in general is not done in a mathematical text. When can one afford such digressions while presenting content? If this book is like a chat between friends, then this secondary story is embedded in the structure of the overall construction, and the mix is not bad at all.

Once this gamble is accepted, one can go directly to Leonhard Euler's classical inequality $R \geq 2r$ (between circumradius and inradius), then move over to more technical inequalities in a triangle, as, e.g.,

$$\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \leq \frac{abc}{8abc} = \frac{1}{8},$$

whose proof involves trigonometry serving the just cause of the advanced Euclidean geometry. Vodă writes: "Euler did not prove his inequality by this method. He loved too much geometry to reduce it to a simple calculation."

This assertion actually is quite the contrary of what Oene Bottema writes in [2], where Euler is suspected of enjoying his "computational desire" too much. The methods used in Vodă's book are all synthetic, he is using no analytic methods in this book, although Euler's original method in [11] was actually of analytic nature, and the local high-school textbook at that time in Romania was using analytic methods to approach certain proofs (e.g. Law of Sines). Even if he argues for the beauty of individual approaches, the author is careful with the consistency of his method.

The detailed discussion on Euler's line theorem [11] is extended and continued with a plethora of theorems and properties either carefully collected from the historical collection of the monthly *Gazeta matematică*, which had a continuous printing from 1895 until today, or from two fundamental sources [13, 14], quite known to the connaisseurs of the old-fashioned geometry, but out of print at that time. The outcome looks like a nice blend of the ideas from [7] and [19] interrupted by well-placed divagations helping the transition from one idea into another.

The third Chapter in the book is dedicated to inequalities (the title is *A Fight Without Any Standing Chance*), and the content could rival any elementary monograph on fundamental inequalities written in that period. Chapter 4 is titled *The Betrayal*, with a metaphor meant to suggest that the book deals not only with triangles, but also with polygons. Vodă starts his discussion with a theorem of L. Carlitz, dedicated to a quadrilateral that is both cyclic and also admits an incircle. Then, in such a configuration, the following relation holds

$$R^2 - OI^2 = 2 \cdot \frac{(ab+cd)(ad+bc)}{(a+c)^2(ac+bd)} Rr,$$

where a, b, c, d are the lengths of the quadrilateral's sides, R is the circumradius, r is the inradius, O is the circumcenter, and I is the incenter. The proof is not difficult, it relies on trigonometry, and the author breaks down the steps patiently.

The book concludes with three quite extended appendices. The first one contains 115 definitions and old-fashioned terms, the second one is dedicated to selected problems in advanced Euclidean geometry, and among the anecdotes recounted there we find out memories from the time when the writer visited the Bay Area and took classes as a visitor at UC Berkeley. The third appendix includes *25 Selected Theorems of Old-Fashioned Geometry*, and the fourth presents no less that 187 formulae of advanced Euclidean geometry, some of them fundamental, while some others less-known and quite exotic.

There Are Some Gaps. From the historical standpoint, there are quite a few lapses in Viorel Gh. Vodă's texts, some due to his historical context and lack of access to original sources. It can be suspected that Vodă never had proper access to Leonhard Euler's collected works. Also, he does not seem to be aware that Count Giulio Carlo di Fagnano [10] is responsible for the original proof of the properties of the orthic triangle, nor he does cite the

original Leonhard Euler's source [11] for several properties related to the first five points of interest in the triangle. Vodă's approach main weakness is that he does not rely on the original sources, quite probably not available to him at that time in the Romanian libraries in the historical period 1980-1982. He refers repeatedly to intermediate sources.

The Author. Viorel Gh. Vodă was born on January 8, 1944, in Bucharest, graduated from the University of Bucharest with a B.Sc. in Mathematics in 1962, then with M.A. in Mathematics in 1967 [29]. He defended his doctoral dissertation titled *Generalized Rayleigh model in durability analysis* at the Center for Mathematical Statistics, with the Romanian Academy, and therefore was a highly qualified statistician by trade. His non-specialist discourse was just a rhetorical tool. Since 1967, Vodă worked at the Romanian Academy Center for Mathematical Statistics, towards the end of his career serving as the head of the Operation Research Department. Viorel Gh. Vodă described his academic profile as focused on distribution theory, quality control, and reliability. Over the years he delivered talks in many academic events around the world, including at UC Berkeley (1974), University of Tampa (1975), Columbia University (1975), and North Carolina University at Raleigh (1989) [29]. Most of his research was published in Romanian journals, e.g., [22, 24, 25, 27], and in the foreword of his book he pays homage to the Romanian political regime at that time, a personal option that may very well be genuine.

Conclusions. Successful presentations depend on their cultural and historical context. We cannot be sure whether a new edition of Vodă's text would attract the same interest in Romania today, or whether its translation would mean anything to the international reader. Triangle geometry fades our of fashion [9] and its content moves towards other directions, e.g., axiomatics, metric geometry, complex analysis or computer sciences. As Shiing-Shen Chern said it once, geometry always changes shape. It is true that irrespective of the culture, a catchy style, written from a humble non-emphatic perspective attracts the reader and stands a much better chance than any high-pedestal grim attitude. Knowing that an interested audience awaits out there, willing to read a text written in an appropriate style, with substantive content, is always a plus, and the author should always have a feeling for his audience's need.

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