Considerations concerning some inequalities of the arithmetic functions $\sigma_k^{(e)}$ and $\tau^{(e)}$

Nicusor Minculete

Abstract: The objective of this paper is to present the inequality

$$\frac{\sqrt{\sigma_k^{(e)}(n)\sigma_l^{(e)}(n)}}{\sigma_{\frac{k+l}{2}}^{(e)}(n)} \le \frac{n^{\frac{l-k}{4}}\sigma_k^{(e)}(n) + n^{\frac{k-l}{4}}\sigma_l^{(e)}(n)}{2\sigma_{\frac{k+l}{2}}^{(e)}(n)} \le n^{\frac{|k-l|}{4}} \le \frac{n^{\frac{k-l}{2}}+1}{2},$$

for every $n, k, l \in N$ with $n \ge 2$ and $\frac{k+l}{2} \in N$, where $\sigma_k^{(e)}(n)$ is the sum of kth powers of exponential divisors of n and $\tau^{(e)}(n)$ - the number of exponential divisors of n.

2000 Mathematics Subject Classification: 11A25

Key words: the sum of kth powers of exponential divisors of n, the number of exponential divisors of n.

Introduction

Let n be a positive integer, $n > 1$. We note $\sigma_k^{(e)}(n)$ the sum of kth powers of exponential divisors of n, so, $\sigma_k^{(e)}(n) = \sum_{d|_{(e)}n} d^k$, whence we obtain the following equalities: $\sigma_1^{(e)}(n) = \sigma^{(e)}(n)$ and $\sigma_0^{(e)}(n) = \tau^{(e)}(n)$ - the number of exponential divisors of n. The notion of *exponential divisors* was introduced by M. V. Subbarao see [1].