

CONCURSUL NAȚIONAL DE MATEMATICĂ APLICATĂ “ADOLF HAIMOVICI”

Faza locală, 12 februarie 2024

Clasa a X-a

H2 – Filiera teoretică - Profil real - Specializarea Științe ale naturii

BAREM DE EVALUARE ȘI NOTARE

Problema 1

$$\begin{aligned} \text{a) } a^3 &= 5\sqrt{2} + 7 - 5\sqrt{2} + 7 - 3\sqrt{(5\sqrt{2} + 7)(5\sqrt{2} - 7)} \left(\sqrt[3]{5\sqrt{2} + 7} - \sqrt[3]{5\sqrt{2} - 7} \right) & 2p \\ &= 14 - 3a \Rightarrow a^3 + 3a - 14 = 0 & 1p \\ \text{b) } a(a^2 + 3) &= 14 \Rightarrow a^2 + 3 = \frac{14}{a} & 2p \\ x &= \log_2 \frac{14}{a} + \log_a a^2 + \log_2 a - \log_2 7 = 3 \in \mathbb{N} & 2p \end{aligned}$$

Problema 2

$$\begin{aligned} a &= \left(\log_{2024} \frac{2}{1} \cdot \frac{3}{2} \cdot \dots \cdot \frac{2024}{2023} \right)^{\log_{2024} \frac{1 \cdot 2}{2 \cdot 3} \cdot \dots \cdot \frac{2023}{2024}} & 1p \\ a &= (\log_{2024} 2024)^{\log_{2024} \frac{1}{2024}} = 1^{-1} = 1 & 2p \\ b &= \frac{1-\sqrt{2}}{1-2} + \frac{\sqrt{2}-\sqrt{3}}{2-3} + \dots + \frac{\sqrt{99}-\sqrt{100}}{99-100} = \frac{1-10}{-1} = 9 & 3p \\ \sqrt[3]{a^b - b^a} &= \sqrt[3]{-8} = -2 & 1p \end{aligned}$$

Problema 3

$$\begin{aligned} \text{a) } f &- \text{ inversabilă } \Leftrightarrow f - \text{ bijectivă } \Leftrightarrow & 2p \\ &y = f(x) \text{ are soluție unică } x = \frac{y+1}{2} & 2p \\ \text{b) } f^{-1}: \mathbb{R} &\rightarrow \mathbb{R}, f^{-1}(y) = x, \text{ unde } y = f(x) \Leftrightarrow f^{-1}(y) = \frac{1}{2}y + \frac{1}{2} & 2p \\ f^{-1}(-1) &= -\frac{1}{2} + \frac{1}{2} = 0 & 1p \end{aligned}$$

Problema 4

$$\begin{aligned} z_2 &= (3 - 4i)(2 - i) \Rightarrow \left(\frac{z_1}{z_2} \right)^2 - \left(\frac{z_2}{z_1} \right)^2 = \left(\frac{1}{2-i} \right)^2 - (2-i)^2 = & 3p \\ &= \frac{1}{3-4i} - (3-4i) = \frac{-72+104i}{25} & 3p \\ \operatorname{Re} \left[\left(\frac{z_1}{z_2} \right)^2 - \left(\frac{z_2}{z_1} \right)^2 \right] &= -\frac{72}{25} & 1p \end{aligned}$$