CLASA A V-A

BAREM DE EVALUARE ŞI DE NOTARE

(1) $a = 18b + 6$ $\boxed{2p}$ $(18b+6) + b = 2039$ $\boxed{2p}$

$b = 107$ $\boxed{2p}$ $a = 1932$ $\boxed{1p}$

(2) $5^p < 809 \Rightarrow p \leq 4$ $\boxed{1p}$

Dacă $p \leq 3$, at. $5^m + 5^n + 5^p < 3 \cdot 5^3 < 653$ (F),

deci $p = 4$. $\boxed{2p}$

(I) $n = 3 \Rightarrow m = 0, 1$ sau 2 $\boxed{3p}$

(II) $n = 2 \Rightarrow m = 1$ $\boxed{1p}$

(3) $b = a+1,\ c = a+2$ $\boxed{1p}$ $N = 8^{2a} + 2 \cdot 4^{3a+3} + 2^{6a+12}$ $\boxed{2p}$

$= 2^{6a} + 2^{6a+7} + 2^{6a+12}$ $\boxed{2p}$ $= 2^{6a}(1 + 2^7 + 2^{12})$

$\boxed{1p}$ $= (2^{3a} \cdot 65)^2$ $\boxed{1p}$

(4) $A = 1 + (1\cdot 2 - 1) + (1\cdot 2\cdot 3 - 1\cdot 2) + (1\cdot 2\cdot 3\cdot 4 - 1\cdot 2\cdot 3) +$

$\ldots + (1\cdot 2\cdot 3\cdots 100 - 1\cdot 2\cdot 3\cdots 99)$ $\boxed{3p}$

$= 1\cdot 2\cdot 3\cdots 100$ (după reduceri) $\boxed{3p}$

$A = B$ $\boxed{1p}$

CLASA A VI-A

BAREM DE EVALUARE ŞI DE NOTARE

1) suplementul $= 180^\circ - a^\circ$ $\boxed{1p}$

suplementul suplementului $= 180^\circ - (180^\circ - a^\circ)$
$= a^\circ$ $\boxed{1p}$

complementul suplementului suplementului $=$
$90^\circ - a^\circ$ $\boxed{2p}$

suplementul complementului suplementului suplementului $= 180^\circ - (90^\circ - a^\circ) = 90^\circ + a^\circ$ $\boxed{2p}$

$(90^\circ + a^\circ) - a^\circ = 90^\circ$ $\boxed{1p}$

2) $\frac{a}{1} = \frac{b}{3} = \frac{c}{4} = \frac{d}{5} = \frac{e}{7} = \frac{f}{10} \overset{not}{=} k$ $\boxed{1p}$

$\Rightarrow a = k,\ b = 3k,\ c = 4k,\ d = 5k,\ e = 7k,\ f = 10k$ $\boxed{2p}$

$a^2 + b^2 + c^2 + d^2 + e^2 = k^2 + 9k^2 + 16k^2 + 25k^2 + 49k^2$
$\boxed{2p}$ $= 100k^2$ $\boxed{1p}$ $= (10k)^2 = f^2$ $\boxed{1p}$

3) $d = (a, b) \Rightarrow a = da_1,\ b = db_1$ $\boxed{1p}$

$[a, b] = da_1b_1$ $\boxed{1p}$ $d(2a_1b_1 + 3) = 18$ $\boxed{1p}$

$2a_1b_1 + 3$ - divizor impar, > 3 al lui 18 $\Rightarrow$

$2a_1b_1 + 3 = 9$ $\boxed{1p}$ $\Rightarrow a_1b_1 = 3 \Rightarrow a_1 = 1,\ b_1 = 3$ sau invers $\boxed{1p}$ şi $d = 2$. $\boxed{1p}$ $\Rightarrow \begin{matrix} a = 2 \\ b = 6 \end{matrix};\ \begin{matrix} a = 6 \\ b = 2 \end{matrix}$ $\boxed{1p}$

4) a) $n + (n+3) = (n+1) + (n+2)$ $\boxed{2p}$ $\Rightarrow$

$A \cup \{n, n+1, n+2, n+3\} = (B \cup \{n, n+3\}) \cup (C \cup \{n+1, n+2\})$ $\boxed{2p}$

b) $\{\boxed{1, 2}, 3, \boxed{4}, 5, 6, \boxed{7}, \boxed{8}, 9, 10, \boxed{11}, \dots\}$
$= \{1, 2, 4, 7, 8, 11, \dots\} \cup \{3, 5, 6, 9, 10, \dots\}$ $\boxed{3p}$

CLASA A VII-A

BAREM DE EVALUARE ȘI DE NOTARE

(1) $2022[x]-2021\{x\}=[x]+\{x\}$ $\boxed{1p}$

$2021[x]=2022\{x\}$ $\boxed{1p}$ $\in[0,2022)$ $\boxed{1p}$

$[x]\in\left[0,\frac{2022}{2021}\right)\cap\mathbb{Z}\Rightarrow[x]=0$ sau 1 $\boxed{2p}$

$[x]=0\Rightarrow\{x\}=0\Rightarrow x=0$ $\boxed{1p}$

$[x]=1\Rightarrow\{x\}=\frac{2021}{2022}\Rightarrow x=1\frac{2021}{2022}$ $\boxed{1p}$

(2) $\sqrt{3+2\sqrt{2}}=\sqrt{2}+1$ $\boxed{2p}$ $\sqrt{6-4\sqrt{2}}=2-\sqrt{2}$ $\boxed{2p}$

Raționalizare: $E=\sqrt{2}(\sqrt{2}-1)+(2-\sqrt{2})(3+2\sqrt{2})$ $\boxed{2p}$

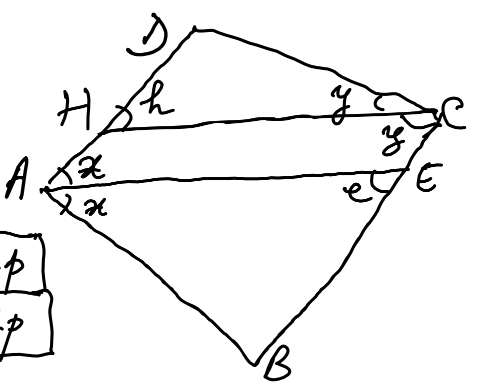
$=2-\sqrt{2}+6+4\sqrt{2}-3\sqrt{2}-4=4$ $\boxed{1p}$

(3) $\sphericalangle AEB\equiv\sphericalangle HCB$

$\Rightarrow e=y$ $\boxed{1p}$

$\sphericalangle CHD\equiv\sphericalangle EAD$

$\Rightarrow h=x$ $\boxed{1p}$



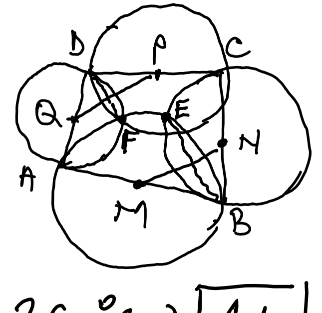
În ΔABE: $m(\sphericalangle B)=180^\circ-x-y$ $\boxed{2p}$

În ΔCHD: $m(\sphericalangle D)=180^\circ-x-y$ $\boxed{2p}$

$\sphericalangle B\equiv\sphericalangle D$ $\boxed{1p}$

(4) a) $BE\perp MN$, $DF\perp PQ$ $\boxed{2p}$

$MN\parallel PQ$ $\boxed{1p}$ $BE\parallel DF$ $\boxed{1p}$



b) Dacă X este un punct comun, atunci $\widehat{AXB}>90^\circ$ etc $\boxed{2p}$

$\Rightarrow\widehat{AXB}+\widehat{BXC}+\widehat{CXD}+\widehat{DXA}>360^\circ$ (F) $\boxed{1p}$

CLASA A VIII A

BAREM DE EVALUARE ŞI DE NOTARE

① $E=\left[x^2+(y-3)x+\left(\frac{y-3}{2}\right)^2\right]+y^2-3y+3-\left(\frac{y-3}{2}\right)^2$ $\boxed{2p}$

$=\left(x+\frac{y-3}{2}\right)^2+\frac{3}{4}(y-1)^2$ $\boxed{2p}$ ≤ 0

$\Rightarrow x+\frac{y-3}{2}=0$ și $y-1=0$ $\boxed{1p}$ $\Rightarrow \begin{matrix} x=1 \\ y=1 \end{matrix}$ $\boxed{2p}$

② $0\le\left(a\pm\frac{1}{2}\right)^2+\left(b\pm\frac{1}{2}\right)^2$ $\boxed{1p}$ $=\frac{3}{2}\pm(a+b)$ $\boxed{1p}$

$-\frac{3}{2}\le a+b\le\frac{3}{2}\Rightarrow a+b\in\{-1,0,1\}$ $\boxed{2p}$

(I) $a+b=-1,\ a^2+b^2=1\Rightarrow a=0,\ b=-1$ sau invers $\boxed{1p}$

(II) $a+b=0,\ a^2+b^2=1\Rightarrow a=\frac{1}{\sqrt{2}},\ b=-\frac{1}{\sqrt{2}}$ sau invers $\boxed{1p}$

(III) $a+b=1,\ a^2+b^2=1\Rightarrow a=0,\ b=1$ sau invers $\boxed{1p}$

③ (VA_iA_j), $1\le i<j\le 5$ sunt 10 plane $\boxed{4p}$

la care se adaugă (P) $\boxed{2p}$, în total 11 plane $\boxed{1p}$

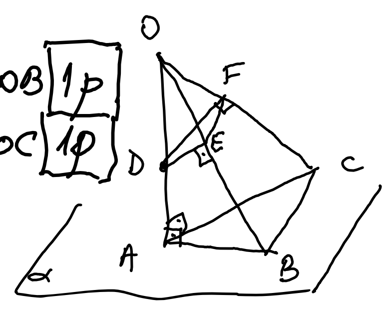
④ $m(\widehat{OAB})=m(\widehat{OAC})=90^\circ$ $\boxed{1p}$

$ABED$ înscr $\boxed{1p}$ $\Rightarrow OD\cdot OA=OE\cdot OB$ $\boxed{1p}$

$ACFD$ înscr $\boxed{1p}$ $\Rightarrow OD\cdot OA=OF\cdot OC$ $\boxed{1p}$

$\Rightarrow OE\cdot OB=OF\cdot OC$ $\boxed{1p}$

$\Rightarrow BCFE$ - înscr $\boxed{1p}$



CLASA A IX A

BAREM DE EVALUARE ȘI DE NOTARE

1) $a\cdot b\in \mathbb{Q}$ $\boxed{1p}$ $a\in \mathbb{R}\setminus\mathbb{Q}$ $\boxed{2p}$ $b\in \mathbb{R}\setminus\mathbb{Q}$ $\boxed{2p}$

$a+b\in \mathbb{R}\setminus\mathbb{Q}$ $\boxed{2p}$

2) $n=0$ $\boxed{1p}$ $49^{n+1}-48(n+1)-1$

$=49\cdot 49^{n}-48n-49$ $\boxed{2p}$

$=49(49^{n}-48n-1)+2304n$ $\boxed{3p}$

Finalizare $\boxed{1p}$

3) $a^2+b^2\geq 2ab \Rightarrow ab\leq \frac{1}{2}$ $\boxed{2p}$

$a+\frac{1}{2a}\geq 2\cdot\sqrt{a\cdot\frac{1}{2a}}=\sqrt{2}$ $\boxed{1p}$

$b+\frac{1}{2b}\geq 2\sqrt{b\cdot\frac{1}{2b}}=\sqrt{2}$ $\boxed{1p}$

$\frac{1}{2a}+\frac{1}{2b}\geq 2\sqrt{\frac{1}{2a}\cdot\frac{1}{2b}}=\frac{1}{\sqrt{ab}}\geq\sqrt{2}$ $\boxed{2p}$

Finalizare prin adunare $\boxed{1p}$

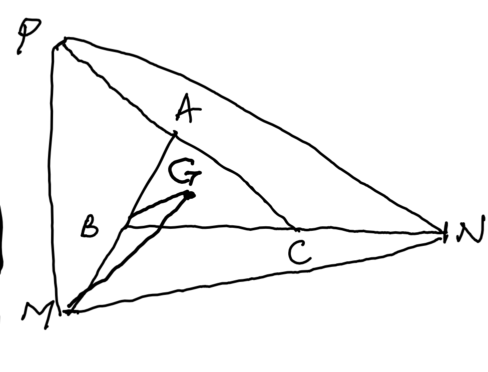
4) $\sum \overrightarrow{GM}$

$=\sum\left(\overrightarrow{GB}+\overrightarrow{BM}\right)$ $\boxed{2p}$

$=\sum \overrightarrow{BM}$ $\left(\sum\overrightarrow{GB}=0\right)$ $\boxed{1p}$

$=\sum \overrightarrow{AB}$ $\left(\overrightarrow{BM}=\overrightarrow{AB}\right)$ $\boxed{2p}$

$=\overrightarrow{0}$ $\boxed{1p}$ Finalizare $\boxed{1p}$



CLASA A X A

BAREM DE EVALUARE ȘI DE NOTARE

① $(7-a\sqrt{2})+(7+a\sqrt{2})+3\cdot 2\cdot\sqrt[3]{(7-a\sqrt{2})(7+a\sqrt{2})}=8$ $\boxed{1p}$

$\Rightarrow 49-2a^2=-1$ $\boxed{2p}$ $\Rightarrow a=\pm 5$ $\boxed{1p}$

Reciproc, $x=\sqrt[3]{7-5\sqrt{2}}+\sqrt[3]{7+5\sqrt{2}}\Rightarrow$

$x^3=14+3\sqrt[3]{(7-5\sqrt{2})(7+5\sqrt{2})}\cdot x$ $\boxed{1p}$

$\Rightarrow x^3+3x=14$ $\boxed{1p}$; x nu e $\lessgtr 2\Rightarrow x=2$, $\boxed{1p}$

② $\omega^2+\omega+1=0$ $\boxed{1p}$

$(2+5\omega+2\omega^2)^6=(2(1+\omega^2)+5\omega)^6=(-2\omega+5\omega)^6$ $\boxed{2p}$

$=(3\omega)^6=3^6(\omega^3)^2=3^6=729$ $\boxed{1p}$

$(2+2\omega+5\omega^2)^6=(2(1+\omega)+5\omega^2)^6=(-2\omega^2+5\omega^2)^6$ $\boxed{2p}$

$=(3\omega^2)^6=3^6(\omega^3)^4=3^6=729$ $\boxed{1p}$

③ $\sum(a+b)\cdot\sum\frac{\log_a b}{a+b}\overset{CBS}{\geq}\left[\sum\left(\sqrt{a+b}\cdot\sqrt{\frac{\log_a b}{a+b}}\right)\right]^2$ $\boxed{3p}$

$=\left(\sum\sqrt{\log_a b}\right)^2\geq\left(3\sqrt[3]{\prod\sqrt{\log_a b}}\right)^2$ $\boxed{2p}$ $=9$ $\boxed{2p}$

④ a) $f(x_1)=f(x_2)\Rightarrow f(f(x_1))=f(f(x_2))\Rightarrow x_1=x_2$ $\boxed{2p}$

$y=f(x)\Leftrightarrow f(y)=f(f(x))=x+2\Leftrightarrow x=f(y)-2$ $\boxed{2p}$

b) Inducție: $f(n)=n+1,\ n\in\mathbb{N}$ $\boxed{2p}$

$f(2022)=2023\Rightarrow f^{-1}(2023)=2022$ $\boxed{1p}$

CLASA A XI A

BAREM DE EVALUARE ȘI DE NOTARE

1) $A^n=\begin{pmatrix}3^{n-1} & 0 & 3^{n-1}\\ 0 & 0 & 0\\ 2\cdot 3^{n-1} & 0 & 2\cdot 3^{n-1}\end{pmatrix}$, $n=1$ $\boxed{1p}$, pasul de ind. $\boxed{2p}$

$3^{n-1}+3^{n+1}=90$ $\boxed{2p}$ $\Rightarrow n=3$ $\boxed{2p}$

2) $\begin{vmatrix}a+1 & b\\ c & d+1\end{vmatrix}=\begin{vmatrix}1-a & -b\\ -c & 1-d\end{vmatrix}$ $\boxed{2p}$

$\Leftrightarrow (a+1)(d+1)-bc=(1-a)(1-d)-bc$ $\boxed{2p}$

$\Leftrightarrow ad+a+d+1-bc=1-a-d+ad-bc$ $\boxed{2p}$

$\Leftrightarrow a+d=0 \Leftrightarrow \operatorname{Tr}A=0$ $\boxed{1p}$

3) $x_n=a^{(-2)^n}, \forall n\in\mathbb{N}$ $\boxed{2p}$

(x_n) convergent pentru $a=1$ $\boxed{1p}$

Dacă $a>1 \Rightarrow x_{2n}=a^{2^{2n}}\to\infty$, NU $\boxed{2p}$

Dacă $a<1 \Rightarrow x_{2n+1}=\left(\frac{1}{a}\right)^{2^{2n+1}}\to\infty$, NU $\boxed{2p}$

4) $\lim\limits_{x\to\infty}\left(\frac{x^2+1}{x+1}-(x-1)\right)=0$ $\boxed{2p}$

$\lim\limits_{x\to\infty}\left(\sqrt{x^2+x+1}-\left(x+\frac{1}{2}\right)\right)=0$ $\boxed{2p}$

$\lim\limits_{x\to\infty}\left(\frac{x^2+1}{x+1}+\sqrt{x^2+x+1}-\left(2x-\frac{1}{2}\right)\right)=0$ $\boxed{2p}$

$m=2,\ n=-\frac{1}{2}$ $\boxed{1p}$

CLASA A XII-A

BAREM DE EVALUARE ŞI DE NOTARE

(1) $x * y = (x+4)(y+4) - 4$ $\boxed{1p}$

$x_1 * \dots * x_n = (x_1+4)\cdots(x_n+4) - 4$ $\boxed{2p}$

$1 * 2 * \dots * n = 5 \cdot 6 \cdot 7 \cdots (n+4) - 4$ $\boxed{1p}$

se divide cu $5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 = 151200$ $(n \geq 6)$ $\boxed{2p}$

Ultimele două cifre: $\dots 00 - 4 = \dots 96$ $\boxed{1p}$

(2) Dacă $H \leq G$, $x \in H$, $y \in G \setminus H \Rightarrow xy \in G \setminus H$

PRA $xy \in H \Rightarrow x^{-1}(xy) \in H \Rightarrow y \in H$ ∞ $\boxed{1p}$

Fie $b \in G \setminus \{a\}$; $a \notin G \setminus \{a\}$ $\boxed{1p}$

$\Rightarrow ab \in G \setminus (G \setminus \{a\})$ $\boxed{2p}$ $\Rightarrow ab = a \Rightarrow b = e$ $\boxed{2p}$

$\Rightarrow G \setminus \{a\} = \{e\} \Rightarrow G = \{e, a\}$ $\boxed{1p}$

(3) $f = \sin x + 3\cos x$, $g = 2\sin x + \cos x$

Căutăm $a, b \in \mathbb{R}$ a.î. $f(x) = a\,g(x) + b\,g'(x)$ $\boxed{1p}$

$a = b = 1$ $\boxed{2p}$

$I = \int \left(1 + \frac{g'(x)}{g(x)}\right) dx$ $\boxed{2p}$ $= x + \ln g(x) + C$

$= x + \ln(2\sin x + \cos x) + C$ $\boxed{2p}$

(4) a) $I_{n-1} + I_{n+1} = \frac{1}{n}$. $\boxed{4p}$

b) $0 < I_{n+1} < I_{n-1} + I_{n+1} = \frac{1}{n}$ $\boxed{2p}$

deci $I_n \to 0$. $\boxed{1p}$