Barem clasa a VII-a
(OLM 2023-etapa locală)

Problema I. (7 puncte)

Utilizând formula radicalilor dublii se obţine $\sqrt{16-2\sqrt{15}}=\sqrt{15}-1$,

$\sqrt{9-6\sqrt{2}}=\sqrt{6}-\sqrt{3}$, $\sqrt{7-2\sqrt{10}}=\sqrt{5}-\sqrt{2}$, $\sqrt{17-2\sqrt{30}}=\sqrt{15}-\sqrt{2}$,

$\sqrt{8+2\sqrt{15}}=\sqrt{5}+\sqrt{3}$, $\sqrt{7-2\sqrt{6}}=\sqrt{6}-1$...**(4p)**

$N=\frac{\sqrt{15}-1-\sqrt{6}+\sqrt{3}-\sqrt{5}+\sqrt{2}}{\sqrt{15}-\sqrt{2}+\sqrt{5}+\sqrt{3}-\sqrt{6}+1}\cdot\left(\sqrt{3}+1\right)^2=\frac{\sqrt{5}(\sqrt{3}-1)-\sqrt{2}(\sqrt{3}-1)+(\sqrt{3}-1)}{\sqrt{5}(\sqrt{3}+1)-\sqrt{2}(\sqrt{3}+1)+(\sqrt{3}+1)}\cdot\left(\sqrt{3}+1\right)^2=$..**(2p)**

$=\frac{(\sqrt{3}-1)(\sqrt{5}-\sqrt{2}+1)}{(\sqrt{3}+1)(\sqrt{5}-\sqrt{2}+1)}\cdot\left(\sqrt{3}+1\right)^2=\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}\cdot\left(\sqrt{3}+1\right)^2=(\sqrt{3}-1)(\sqrt{3}+1)=2\in\mathbb{N}$..**(1p)**

Problema II. (7 puncte)

$\frac{m+n}{mn}=\frac{1}{6\cdot 337}$ ....**(1p)**

$\frac{m}{337}=6\left(\frac{m}{n}+1\right)$..**(2p)**

Analog $\frac{n}{337}=6\left(\frac{n}{m}+1\right)$..**(2p)**

Atunci $\left(\frac{m}{337}-6\right)\left(\frac{n}{337}-6\right)=\left(6\left(\frac{m}{n}+1\right)-6\right)\left(6\left(\frac{n}{m}+1\right)-6\right)=36\frac{m}{n}\cdot\frac{n}{m}=36$..................................**(1p)**

și $\sqrt{\left(\frac{m}{337}-6\right)\left(\frac{n}{337}-6\right)+1}=\sqrt{37}$ care este număr irațional ....**(1p)**

Problema III. (7 puncte)

a) $\sqrt{\left(3-2\sqrt{3}\right)^2}+\sqrt{\left(\sqrt{3}-3\right)^2}-\sqrt{\left(\sqrt{3}+4\right)^2}+\sqrt{(-4)^2}=\left|3-2\sqrt{3}\right|+\left|\sqrt{3}-3\right|-\left|\sqrt{3}+4\right|+|-4|=$

$=-3+2\sqrt{3}+3-\sqrt{3}-\sqrt{3}-4+4=0\in\mathbb{N}$...**(3p)**

b) $A=\frac{1}{3}\left(\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\cdots+\frac{1}{99\cdot100}\right)=\frac{1}{3}\left(\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{99}-\frac{1}{100}\right)=\frac{1}{3}\left(\frac{1}{1}-\frac{1}{100}\right)=\frac{1}{3}\cdot\frac{99}{100}=\frac{33}{100}$........**(2p)**

$B=\sqrt{\frac{1}{7}+\left(\frac{9}{14}-\frac{1}{2}\right)+\left(\frac{10}{21}-\frac{1}{3}\right)+\cdots+\left(\frac{70}{441}-\frac{1}{63}\right)}=\sqrt{\frac{1}{7}\cdot 63}=3$ ....**(1p)**

$A\cdot B+\frac{1}{100}=\frac{99}{100}+\frac{1}{100}=1$ ....**(1p)**

Problema IV. (7 puncte)

Desen corect...**(1p)**

ΔADE isoscel, $\sphericalangle DAE=90°+60°=150°\Rightarrow\sphericalangle ADE=\sphericalangle AED=15°$..**(1p)**

ΔAEB echilateral, $\sphericalangle AEB=60°\Rightarrow\sphericalangle DEB=45°$..**(2p)**

ΔDEF echilateral, $\sphericalangle DEF=60°\Rightarrow\sphericalangle BEF=15°$..**(2p)**

$\Delta ADE\equiv\Delta BEF(LUL)\Rightarrow AD=BF=BE\Rightarrow\Delta BEF$ isoscel $\Rightarrow\sphericalangle BFE=15°$...**(1p)**