MATHMOISELLE: Mathematics Contest in Modern Languages, VIIth edition, 6/2/2016

9thclass/M$_1$

First and last name of the student

Calculus problems

1. If $(a_n)_{n\in N}$ is an arithmetic progression where $a_1 + a_{2017} = 2017$, then a_{1009} is equal to:

$a)2017$ $\quad b)1009$ $\quad c)\frac{2017}{2}$ $\quad d)\frac{1009}{2}$

2. The product of the solutions to this equation $\left[\frac{x-1}{2}\right] = \frac{x+3}{3}$ is:

a) 135 b)9 c)108 d) 1620

3. Let be the set $M = \{\cos 1^\circ \cdot \sin 1^\circ,\ \cos 3^\circ \cdot \sin 3^\circ,\ ...,\ \cos 179^\circ \cdot \sin 179^\circ\}$. If a is the smallest and b the largest element of the set, then these elements are:

a) $a = b = 0$ *b)* $a = -\frac{1}{2}, \mathrm{b} = \frac{1}{2}$ *c)* $a = -1, b = 1$ *d)* $a = -\frac{\sqrt{2}}{2}, \mathrm{b} = \frac{\sqrt{2}}{2}$

Logical problems

1. The number of the diagonals of a convex polygon with 2016 vertices is:

$a)1008 \cdot 2013$ $\quad b)1008$ $\quad c)2016 \cdot 2015$ $\quad d)2016 \cdot 2013$

2. Cryptology is the science of secret writings and its objective is to keep secret both data and classified information by means of cryptographic systems. One of the simplest systems is based on the cipher of Caesar (the renowned Roman emperor): the clear text is built using the letters of the Latin alphabet *A,B,...,Z* and the *coding key* is represented by a whole number $k \in \{1,2,3,...,26\}$. A lexicographic order x is associated to each letter in the source text, then the named order is replaced by the code character $(x+k) \bmod 26$ (as $\bmod 26$ is considered the rest of the division of $x+k$ by 26). For instance, the word *MATEMATICA* is coded using the algorithm with the key $k = 9$ in the following manner: $x = 13$ corresponds to the letter *M*, so one should code it into $(13+9) \bmod 26 = 22$ and then continue in this manner until obtaining *VJCNVJCRLJ*. Using the algorithm with the key $k = 10$, code the message *MATHMOISELLE*.

a) VLOAVYCRUJJU b) WKDRWYSCOVVO c) ZWTSZJIASDDS d) WKDRYWSCOVVO

3. Given the predicate $p(x,y): x^2 - 20x + 100 + y^2 = 0, x, y \in R$ find out which of the following sentences is true:

a) $(\exists)\mathrm{x}, (\exists)\mathrm{y}, \mathrm{p}(\mathrm{x},\mathrm{y})$ b) $(\forall)\mathrm{x}, (\forall)\mathrm{y}, \mathrm{p}(\mathrm{x},\mathrm{y})$
c) $(\forall)\mathrm{x}, (\exists)\mathrm{y}, \mathrm{p}(\mathrm{x},\mathrm{y})$ d) $(\exists)\mathrm{x}, (\forall)\mathrm{y}, \mathrm{p}(\mathrm{x},\mathrm{y}$

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9thclass/M$_1$

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Practical applications

1. Imagine a cube whose edge is 3 m long, divided into 27 identical small cubes. An ant wants to pass over all the smaller cubes, except for the one in the middle, by crossing only the edges between adjoining cubes, and avoiding the points. The maximum number of small cubes the ant can cross in this manner is:

$a)26 \quad b)25 \quad c)27 \quad d)23$

2. Dan is watching a flock of birds. Each minute, some birds depart from the flock and others join it. Dan counts them each time and finds a rule. The largest natural number which is smaller than 10% from the overall number of birds depart and 5 others come to join the flock. Taking into account that in the beginning the flock had 20 birds, Dan says that in 10 minutes there will be

a) 40 *b)* 42 *c)* 39 *d)* 43

3. In the String Theory, our Universe is placed on an infintely long but narrow membrane, in such a way that it can be compared to a rope, and this membrane is situated between a multitude of other such parallel membranes. Some scientists consider that these membranes vibrate according to certain laws and that the collision between the membrane containing our Universe and that of a parallel universe caused the Big Bang. Which of the following functions $f:[0,\infty)\to R$ can be the one causing to vibrate the membrane that contains our universe?

a) $f(x)=\cos x$ b) $f(x)=2x+1$ c) $f(x)=\{x\}$ d) $f(x)=[x]$

MATHMOISELLE: Mathematics Contest in Modern Languages, VIIth edition, 06/02/2016

9thclass/M2

First and last name of the student

Calculus problems

1. If we calculate $S = \frac{1}{1\cdot 5} + \frac{1}{5\cdot 9} + \ldots + \frac{1}{(4\cdot 2016-3)\cdot(4\cdot 2016+1)}$ we will obtain:

$a)\frac{2015}{2016}$ $\quad b)\frac{1}{8061}$ $\quad c)\frac{8064}{8061}$ $\quad d)\frac{2016}{8065}$

2. The function $f:\mathbb{R} \to \mathbb{R}$, which verifies the relationship:
$3f(x) - 2f(2-x) = 10x - 7, (\forall)\, x \in \mathbb{R}$, is:

a) $f(x) = 10x - 7$ b) $f(x) = 7 - 10x$ c) $f(x) = 2x + 1$ d) $f(x) = 3x - 2$

3. The result of the calculus $(1+2)\cdot(1+2^2)\cdot(1+2^4)\cdot\ldots\cdot\left(1+2^{2^{10}}\right)$ is:

a) $2^{2^{20}}+1$ $\quad$ *b)* $2^{2^{20}}-1$ $\quad$ *c)* $2^{2^{11}}+1$ $\quad$ *d)* $2^{2^{11}}-1$

Logical problems

1. Emil has tested his new car. He rode the distance of 60 km between Bucharest and Ploiesti at an average speed of 60 km/h, and the 240 km from Ploieşti to Sibiu at an average speed of 80 km/h. The average speed at which he travelled from Bucureşti to Sibiu was:

$a)75\ km/h$ $\quad b)72{,}5 km/h$ $\quad c)80 km/h$ $\quad d)73{,}75 km/h$

2. Andrei has been watching a flight of pigeons. Every minute 10% of the pigeons leave the flight and 30% of them return to the flight. If p_n represents the number of pigeons at the end of an ***n*** minute, then the recurrence relation is:

a) $p_{n+1} = 1{,}2p_n$ $\qquad$ b) $p_{n+1} = 0{,}5p_n + 20$

c) $p_{n+1} = 0{,}9p + 2$ $\qquad$ d) $p_{n+1} = 0{,}8p_n$

3. Consider the sets: $S_1 = \{1\}, S_2 = \{2,3\}, S_3 = \{4,5,6\}, S_4 = \{7,8,9,10\}\ldots$, and so on. Which is the greatest element in set S_{2016} ?

$a)1008\cdot 2017$ $\quad b)201600$ $\quad c)1008$ $\quad d)1008\cdot 2015$

Practical applications

1. An amphitheatre has 39 seats in the second row, 42 seats in the third one and so on. If the number of seats on the rows are in arithmetic progression, find out how many seats there are in the 17th row of the amphitheatre.

a) 102 ; *b)* 104 ; *c)* 84 ; *d)* 95

2. Cryptology is the science of secret writings and its objective is to keep secret both data and classified information by means of cryptographic systems. One of the simplest systems is based on the cipher of Caesar (the renowned Roman emperor): the clear text is built using the letters of the Latin alphabet *A,B,...,Z* and the *coding key* is represented by a whole number $k \in \{1, 2, 3, \ldots, 26\}$. A lexicographic order x is associated to each letter in the source text, then the named order is replaced by the code character $(x+k) \bmod 26$ (as $\bmod 26$ is considered the rest of the division of $x+k$ by 26). For instance, the word *MATEMATICA* is coded using the algorithm with the key $k=9$ in the following manner: $x=13$ corresponds to the letter *M*, so one should code it into $(13+9) \bmod 26 = 22$ and then continue in this manner until obtaining *VJCNVJCRLJ*. Using the algorithm with the key $k=10$, code the message *MATHMOISELLE*.

a) *VLOAVYCRUJJU* *b)* *WKDRWYSCOVVO* *c)* *ZWTSZJIASDDS* *d)* *WKDRYWSCOVVO*

3. Mihai submits one of this works to an exhibition. In order to do this, he takes a wooden disc and cuts it into 12 sectors whose areas are in an arithmetical progression. Then he places the resulting sectors one on top of the other in decreasing order. He notices that the area of the largest sector represents the double of the smallest sector and wonders which could be the measure, in degrees, of the angle corresponding to the smallest sector.

a) 30° *b)* 20° *c)* 10° *d)* 15°.