Mathematics Contest in Modern Languages, VIIth edition, 2.06.2016

11thclass/M$_1$

First and last name of the student

Calculus problems

1. The result of $\lim_{n\to\infty}\left[\lim_{x\to 0}\left(1+\sin^2 x+\sin^2 2x+...+\sin^2 nx\right)^{\frac{1}{x^2}}\right]^{\frac{1}{n^3}}$ is:

   a) $\sqrt{e}$ *b)* $\sqrt[3]{e}$ *c)* ∞ *d)* nu există limita.

2. Let $A=\begin{pmatrix}1 & -1\\ 1 & 3\end{pmatrix}\in M_2(R)$. For $n\in N^*$, A^n equals :

   a) $2^{n-1}\begin{pmatrix}-n+2 & -n\\ n & n+2\end{pmatrix}$ *b)* $2^n\begin{pmatrix}-n+2 & -n\\ n & n+2\end{pmatrix}$ *c)* A *d)* I_2.

3. Let $A=\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{pmatrix}\in M_3(R)$. The rank of $A^{2017}+A^{2018}+I_3$ is:

   a)0 *b)* 1 *c)* 3 *d)* 2.

Logical problems

1. Let us consider the sequence $(a_n)_{n\geq 1}$ cu $a_n=3+(-1)^n$. The value of $\lim_{n\to\infty}\left(\frac{a_1+a_2+...+a_n}{n}\right)$ is:

   a)3 *b)* 4 *c)* 2 *d)* 5.

2. Let $A=\begin{pmatrix}-1 & -1\\ -\varepsilon & -\varepsilon\end{pmatrix}\in M_2(C)$, where $\varepsilon^2+\varepsilon+1=0$. Then, A^{2014} is equal to:

   a) εA *b)* $\varepsilon^2 A$ *c)* O_2 *d)* A.

3. Let us consider the determinant $\Delta=\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^3 & b^3 & c^3\end{vmatrix}$. For any whole values of a,b and c, Δ is divisible by:

   a)5 b) 7 c) 6 d) 10.

Practical problems

1. Lost in the mountains, Mihai, George and Andrei are compelled to set up their improvised tent to spend the night and keep the rain away. The basis of the tent is an equilateral triangle, and the camping site does not help very much. It is rocky and they are unable to thrust the stakes for setting the corners of the tent at will. Mihai finds a small grassy area, and George some less rocky soil. On a piece of paper they draw a sketch, so that the point chosen by Mihai is shown as $M(0,2)$ and the spot found by George as an axis Ox belonging to a Cartesian coordinates devised by them. When

Mihai is placed in point *M*, and George moves along his line Ox, Andrei has to look for his vertex of the tent (the third vertex from the bottom of the equilateral triangle) on :
a) a line *b)* a point *c)* the point can not be found *d)* the union of two lines.
You help him!

2. Suppose that the diameter of an animal's pupils is given by $f(x)$ mm, where x is the intensity of light on the pupils. If $f(x)=\frac{160x^{-0,4}+90}{4x^{-0,4}+15}$, m is the diameter of the pupils with minimum light and M is the diameter of the pupils with maximum light, then m and M are:
a) m = 40 mm, M = 6 mm, b) m = 0.6 mm, M = 4 mm c) m = 8 mm, M = 15 mm
d) m = 14 mm, M = 24mm.

3. Mircea and Vlad have a hobby: secret codes, encoding and decoding messages. To exercise their hobby, they need an invertible matrix $A\in M_n(Z)$. The higher n is, the more complex the encoding of the messages becomes. This time, to simplify the calculations, they will choose a matrix $A\in M_3(Z)$, $A=\begin{pmatrix}6&5&2\\5&5&2\\2&2&1\end{pmatrix}$, called coding matrix. Then, they associate a number to each letter in the alphabet, as well as to all the spaces and question and exclamation marks, as it follows:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Z		?	!
26	27	28	29

They choose the message to be coded and, using the above-mentioned correspondence, they appoint a number for every letter in the chosen message. However, in order to make calculations easier, they will eliminate the spaces between words.

I	M	A	G	I	N	A	T	I	O	N	I	S	M	O	R	EIMPORTANTTHANKNOWLEDGE
9	13	1	7	9	14	1	20	9	15	14	9	19	13	15	.....	

They arrange the numbers vertically in a three-line matrix. The resulting matrix is $\begin{pmatrix}9&7&1&1519\\13&9&201413\\1&14&9&9&15\end{pmatrix}\in M_{3\times5}(Z)$.

They multiply the coding matrix *A* with the resulting one: $\begin{pmatrix} 6 & 5 & 2 \\ 5 & 5 & 2 \\ 2 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 9 & 7 & 1 & 15 & 19 \\ 13 & 9 & 20 & 14 & 13 \\ 1 & 14 & 9 & 9 & 15 \end{pmatrix} = \begin{pmatrix} 121 & 115 & 34 & 178 & 209 \\ 112 & 108 & 33 & 163 & 190 \\ 45 & 46 & 15 & 67 & 79 \end{pmatrix} \in M_{3\times 5}(Z)$ then they divide the resulting 15 numbers by 29 and form a new matrix only with the resulting rests: $\begin{pmatrix} 5 & 28 & 5 & 4 & 6 \\ 25 & 21 & 4 & 18 & 16 \\ 16 & 17 & 15 & 9 & 21 \end{pmatrix}$. Now, associating each figure the corresponding letter, they obtain the following $\begin{pmatrix} 5 & 28 & 5 & 4 & 6 \\ 25 & 21 & 4 & 18 & 16 \\ 16 & 17 & 15 & 9 & 21 \end{pmatrix} \to \begin{pmatrix} E & ? & E & D & F \\ Y & U & D & R & P \\ P & Q & O & I & U \end{pmatrix}$, and the encoded message is EYP?UQEDODRIFPU.

In order to decode the message, the numbers are arranged likewise, vertically, in a matrix. Then they determine the inverse of the coding matrix. The values in the result matrix are then divided by 29, in accordance to the division with remainder theorem (the dividend = the divisor x the quotient + the remainder, where the remainder is always a positive number, necessarily lower than the divisor!)

Now, Mircea and Vlad would like to ask you to encode the last nine letters of the message and to decode the following nine-letter word, using the same encoding matrix: ANSK UKJJ. The two messages will be:
a) UJGEKOKG , GEOMETRIE *b)* KVBJ!AFXE, PRIMAVARA *c)* UJGEKOKG , PRIMAVARA
d) KVBJ!AFXE, GEOMETRIE.

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11thclass/M_2

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Calculus problems

1. The result of $\lim\limits_{x\to\infty}\dfrac{[x]+[2\cdot x]+\ldots+[14\cdot x]}{x}$ is:

   a) 105 *b)* 0 *c)* ∞ *d)* the limit does not exist.

2. The pair of real numbers (x,y,z) for which we have the sum $A+{}^tA=\begin{pmatrix}2&4&6\\x&-2&0\\y&z&2\end{pmatrix}$, $A\in M_3(R)$ is:

   a) (2, 4, 6) *b)* (6, 4, 2) *c)* (4, 6, 0) *d)* (4, 0, 6).

3. The result of $\begin{vmatrix}1-2^{2016}-2016^2&\sqrt{2016}&\sqrt{2016}\\2^{2016}&1-2016^2-\sqrt{2016}&2^{2016}\\2016^2&2016^2&1-2^{2016}-\sqrt{2016}\end{vmatrix}$ is:

   a) $\left(1+2^{2016}+2016^2+\sqrt{2016}\right)^2$ *b)* $\left(1-2^{2016}-2016^2-\sqrt{2016}\right)^2$ *c)* $2^{2016}\cdot 2016^2\cdot\sqrt{2016}$ *d)* 0.

Logical problems

1. The values $a,b\in R$ so that $\lim\limits_{x\to\infty}\left(\dfrac{x^2+1}{x+1}-2ax-3b\right)=1$ are:

   a) $a=\dfrac{1}{2},b=\dfrac{1}{3}$ *b)* $a=-\dfrac{1}{2},b=\dfrac{2}{3}$ *c)* $a=-\dfrac{1}{2},b=-\dfrac{1}{3}$ *d)* $a=\dfrac{1}{2},b=-\dfrac{2}{3}$.

2. Let $A=\begin{pmatrix}1&0\\0&2\end{pmatrix}\in M_2(R)$. Then $\sum\limits_{k=1}^{2016}A^k$ equals to:

   a) $\begin{pmatrix}2016&0\\0&4032\end{pmatrix}$ *b)* $\begin{pmatrix}2016&0\\0&2^{2016}\end{pmatrix}$ *c)* $\begin{pmatrix}2016&0\\0&2^{2017}-2\end{pmatrix}$ *d)* $\begin{pmatrix}2016&0\\0&2^{2016}-1\end{pmatrix}$.

3. If $M=\sum\limits_{a\in A}a^2$, where $A=\left\{a\in R\,|\,B^*=B^{-1},B=\begin{pmatrix}0&a&-a\\-3&5&-3\\-a&a&0\end{pmatrix}\right\}$, then:

   a) $M=0$ *b)* $M=2$ *c)* $M=1$ *d)* $M=4$.

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Practical problems

1. A tank contains 10 liters of pure water. Salty water containing 20 grams per liter is pumped into the tank at a speed of 2 liters per minute. The formula for salt concentration *C(t)* (in g/L) after *t* minutes as well as the long-term concentration of salt, i.e., $\lim_{t\to\infty} C(t)$ will be:

a) $C(t)=\frac{20t}{5+t}, \lim_{t\to\infty} C(t)=20$ *b)* $C(t)=\frac{100t}{5+20t}, \lim_{t\to\infty} C(t)=5$ *c)* $C(t)=\frac{10t}{1+20t}, \lim_{t\to\infty} C(t)=\frac{1}{2}$

d) $C(t)=\frac{20t}{1+10t}, \lim_{t\to\infty} C(t)=2$.

2. Three workers have made together 2064 components. The first worker produced 140% of what the second one did. Also, 60% of what the second worker produced is 15% more than 25% of the third worker's production. How many components did the second worker make?

a) 460 b) 644 c) 504 d) 960

2. Suppose Andrei and George open a fast food restaurant, making 3 different types of sandwiches to sell: *(1)* with bread, butter and jam, *(2)* with bread, ham, cheese, and *(3)* with bread, cheese and sauces. Each morning, Andrei *(A)* and George *(G)* plan to sell a number from each kind as shown in the matrix *M*, $M=\begin{array}{c} \\ A \\ G \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 4 & 5 & 3 \\ 3 & 3 & 6 \end{pmatrix} \end{array} \in M_{(2\times 3)}(N)$.

Matrix *N* describes the number of slices of bread *(p)*, ounces of butter *(u)*, ounces of jam *(d)*, slices of ham *(s),* slices of cheese *(c),* and spoons of sauces *(m)*The necessary ingredients for all these are represented in another matrix

$N=\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccccc} p & u & d & s & c & m \end{array} \\ \begin{pmatrix} 2 & 4 & 40 & 0 & 0 \\ 2 & 0 & 03 & 2 & 0 \\ 2 & 0 & 00 & 4 & 2 \end{pmatrix} \end{array} \in M_{(3\times 6)}(N)$. In order to determine the necessary ingredients for the sandwiches in matrix *M*, George makes the following calculus: 3(2) + 3(2) + 6(2) = 24 slices of bread (*p*), 3(4) + 3(0) + 6(0) = 12 ounces of butter (*u*), 3(4) + 3(0) + 6(0) = 12 ounces of jam (*d*) etc., and the result is to be found in matrix

$P = \begin{matrix} \\ A \\ G \end{matrix}\begin{matrix} \begin{matrix} p & u & d & s & c & m \end{matrix} \\ \begin{pmatrix} 24 & 16 & 1615 & 22 & 6 \\ 24 & 12 & 129 & 30 & 12 \end{pmatrix} \end{matrix} \in M_{(2\times 6)}(N)$. The cost of the ingredients used is shown in matrix $C = \begin{matrix} \\ 1 \\ 2 \\ 3 \end{matrix}\begin{matrix} \begin{matrix} p & u & d & s & c & m \end{matrix} \\ \begin{pmatrix} 2 & 4 & 40 & 0 & 0 \\ 2 & 0 & 03 & 2 & 0 \\ 2 & 0 & 00 & 4 & 2 \end{pmatrix} \end{matrix}\begin{matrix} \begin{matrix} Cost \end{matrix} \\ \begin{matrix} p \\ u \\ d \\ s \\ c \\ m \end{matrix}\begin{pmatrix} 0,12 \\ 0,15 \\ 0,10 \\ 0,30 \\ 0,25 \\ 0,12 \\ \end{pmatrix} \end{matrix} = \begin{matrix} \\ 1 \\ 2 \\ 3 \end{matrix}\begin{matrix} \begin{matrix} Cost \end{matrix} \\ \begin{pmatrix} 1,24 \\ 1,64 \\ 1,84 \end{pmatrix} \end{matrix}$.

Hereby, Andrei and George would like to know how much money to budget in order to purchase the ingredients to make all the sandwiches listed in matrix *M.*

*a)*A= 18,46; G = 19,25 *b)*A= 17,60; G = 17,52 *c)*A = 18,46; G = 19,25 *d)* A = 16,50; G = 19.

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