Which Algebra Problems Are Suitable for Highly Gifted Students in Grades 5-6?

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- Mathematical Circles are a form of outreach that bring mathematicians together with K-12 students.
- These students, and sometimes their teachers or parents, meet together with a mathematician, undergraduate, or graduate student in an informal setting to work on interesting topics or problems in mathematics.
- These informal meetings are held outside of school hours, either at night or on the weekends.

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Additionally we covered problems from:

- Abacus International Challenge (students in grades 2-4 and students in grades 5-6)
- American Mathematical Competition (AMC 8, AMC 10, AMC 12, AIME, USAMO)
- Math Kangaroo
- Monographs published in the MSRI's Math Circle series

Mathematical circles or clubs may have the goal of getting students to do well in mathematical competitions.

We are mostly interested in student's enrichment at the Fullerton Mathematical Circle. There is a continuous feedback process and we listen to students' suggestions. We have organized so far four sessions when the students were the speakers.

Statement of the questions discussed in our presentation: **Many gifted students are facing certain challenges while solving algebra problems.**

We see these challenges if we study the empirical data published by MAA from the AMC 8 or AMC 10. How can we, as math educators, address these challenges?

We begin our exploration with the following problem from the 2006 AMC 10 A.

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Which of the following describes the graph of the equation $(x+y)^2 = x^2 + y^2$?

Distractors	**Answer Distribution (in percent)**
the empty set	15.92
one point	13.73
two lines	**18.03**
a circle	8.44
the entire plane	7.78
Omitted	**36.08**

Another problem for which we have empirical data: problem 9 on the 2007 AMC 10 A examination.

Real numbers a and b satisfy the equations $3^a = 81^{b+2}$ and $125^b = 5^{a-3}$. What is the value of ab?

Distractors	**Answer Distribution (in percent)**
-60	4.74
-17	2.92
9	3.02
12	7.62
60	**29.97**
Omitted	**51.70**

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Thus: there are algebraic manipulations that present a challenge even for the usually well-trained students taking AMC 10.
How can the students practice such manipulations on non-standard algebra problems? This is how we turn to our literature for other examples.
Where can we find approachable training problems?

Example of problems from *Gazeta matematică*:
(December 2012 issue)
S: E12.662. (Corina Constantin) Show that

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S: E12.665. (Cornel Țichindean) (a) Show that the number $a = 2010^2 + 2010 + 2011$ is a perfect square.
(b) Determine the numbers of the form $11 \cdot \overline{x0y}$ which are perfect squares.

S:E12.673. (Corina Constantin) Compare the numbers:

$$a = 2 \cdot 2^2 \cdot 2^3 \cdot \ldots \cdot 2^{63}$$

and

$$b = 1 + 2 + 2^2 + \ldots + 2^{2012}.$$

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B.893. Out of 40 shoppers, 12 of them liked the apples, 23 liked the oranges, and twice as many did not like either as many liked both. How many shoppers liked both?

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B.894. If you read a two-digit number backwards, you get a 4.5 times greater number. Find this number.

B.904. The average height of 15 children is 130 cm. However, it turned out that the nurse did not measure Pete's height, instead she measured John's height twice. If she did, the average height would have been 136 cm. How much taller is Pete than John?

Such problems give students the chance to practice in grades 5-6 algebraic manipulations that could become a challenge, as we have seen from the data published by MAA about previous AMC competitions.

Our approach is to expose the 5th and 6th grade gifted students to problems from Abacus International Challenge and *Gazeta matematică*. Each of these programs issue a new set of problems once a month.

What do we learn from these problems?

- As undergraduate students, for some of us the Math Circle sessions are among our first experiences as teachers
- We are seeing at work extremely gifted students
- We develop our teaching skills and we learn how to present to an audience that looks forward to be challenged by interesting problems
- We understand that education of gifted students is a subject of interest in mathematical education

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