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On the co-strong perfectness of the normal product of graphs

by
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Dedicated to Professor Ioan Tomescu on the occasion of his 80th birthday

Abstract

A graph G is strongly perfect if every induced subgraph H has an independent set meeting all the maximal cliques of H [4]. If both G and its complement are strongly perfect, then G is a co-strongly perfect graph. Co-strongly perfect graphs were first studied in [22].

In this paper we present a number of necessary/sufficient conditions concerning the co-strong perfectness of the normal product of graphs.

Key Words: Perfect graph, strongly perfect graph, normal product of graphs.
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1 Introduction

Throughout this paper $G = (V, E)$ is a simple (i.e., a finite, undirected, loopless and without multiple edges) graph with vertex set $V = V(G)$ and edge set $E = E(G)$. By $\overline{G}$ is denoted the complement of G. If $e = xy \in E$, we also write $x \sim y$, and $x \nsim y$ whenever x, y are not adjacent in G.

If $A \subseteq V$, then $G[A]$ is the subgraph of G induced by $A \subseteq V$; shortly, by $H \subseteq G$ we mean that H is an induced subgraph of G, including G itself. By $G - W$ we denote the graph $(V, E - W)$, whenever $W \subseteq E$. By (A, B) we mean the set $\{ab : a \in A, b \in B, ab \in E\}$, where $A, B \subset V, A \cap B = \emptyset$, and we write $A \backsim B$ whenever $ab \in E$ holds for every $a \in A$ and $b \in B$.

P_n, C_n and K_n denote a chordless path on $n \geq 3$ vertices, the chordless cycle on $n \geq 3$ vertices, and the complete graph on $n \geq 1$ vertices, respectively.

Recall that a P_4-free graph is called a cograph*,* while a *chordal* graph is one having no induced C_k for $k \geq 4$.

A *stable (*or *independent)* set in G is a set of mutually non-adjacent vertices, and the *stability number* $\alpha(G)$ of G is the maximum cardinality of a stable set, while $\omega(G) = \alpha(\overline{G})$.

Let $\mathcal{S}(G)$ denote the family of all maximal (with respect to set inclusion) stable sets of G, and

$$\mathcal{S}_\alpha(G) = \{S : S \in \mathcal{S}(G), |S| = \alpha(G)\}.$$

A *clique* in G is a subset A of $V(G)$ that induces a complete subgraph in G. Let $\mathcal{C}(G)$ denote the family of all maximal (with respect to set inclusion) cliques of G. Clearly,

$$\mathcal{C}(G) = \mathcal{S}(\overline{G}), \quad \mathcal{C}_\omega(G) = \mathcal{S}_\alpha(\overline{G}),$$

$$\mathcal{S}_\alpha(G) \subseteq \mathcal{S}(G), \quad \mathcal{C}_\omega(G) \subseteq \mathcal{C}(G)$$

hold for every graph G.

The *chromatic number* and the *clique covering number* of G are denoted by $\chi(G)$ and $\theta(G)$, respectively. The minimum number of cliques that cover all the edges of G is called the line-clique cover number of G and is denoted by $\theta_1(G)$. It is easy to see that $\theta(G) \leq \theta_1(G)$.

A graph G is called perfect if $\chi(H) = \omega(H)$ holds for each induced subgraph H of G [2]. Some basic structural properties of perfect graphs are presented in [14]. The Perfect Graph Theorem states that an undirected graph is perfect if and only if its complement graph is also perfect [10].

If the equality $\alpha(H) = \theta_1(H)$ is true for every induced subgraph H of G, then G is called a θ_1-perfect graph [5].

A graph G is said to be *trivially perfect* if $\alpha(H) = |\mathcal{C}(H)|$ is valid for each subgraph H of G [7].

Theorem 1. *If G is a graph, then the following statements are equivalent:*
(i) *G is θ_1-perfect;*
(ii) *G is trivially perfect;*
(iii) *G is (P_4 and C_4)-free;*
(iv) *$uv \in E(G)$ if and only if $N[u] \subseteq N[v]$ or $N[v] \subseteq N[u]$.*

Proof. The equivalence *(i)* $\Leftrightarrow$ *(iii)* was stated in [5], while the equivalence *(ii)* $\Leftrightarrow$ *(iii)* was proved in [7]. In [1] was shown that *(iii)* $\Leftrightarrow$ *(iv)*. $\square$

Clearly, (P_4 and C_4)-free graphs coincide with P_4-free chordal graphs.

In what follows, we call *t-perfect* every graph that satisfies Theorem 1.

Each t-perfect graph is also perfect (e.g., see [7]), while the converse is not necessarily true (for instance, P_5 is perfect, but not t-perfect).

A set $T \subseteq V(G)$ is called a stable (complete) transversal of G if

$$|T \cap A| = 1 \text{ for every } A \in \mathcal{C}(G), \ (A \in \mathcal{S}(G), \text{ respectively}).$$

In [15] it was shown that if T is a stable (complete) transversal of G, then $T \in \mathcal{S}(G)$, ($T \in \mathcal{C}(G)$, respectively).

It is worth mentioning the following alternative definition of graph perfectness: a graph G is *perfect* if for every induced subgraph H of G, the family $\mathcal{S}_\alpha(H)$ has a complete transversal (or, equivalently, for every subgraph H of G, the family $\mathcal{C}_\omega(H)$ has a stable transversal).

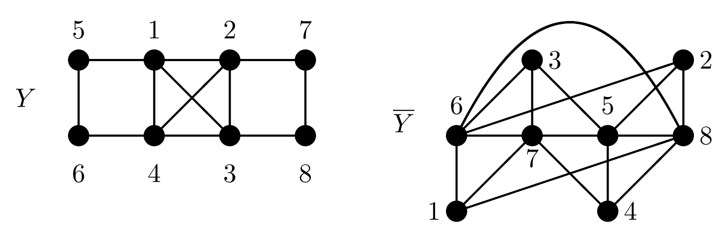

Figure 1: The graph Y and its complement $\overline{Y}$.

A graph G is called c-strongly (s-strongly) perfect if every $H \subseteq G$ has a stable (complete, respectively) transversal. Let us mention that c-strongly perfect graphs are also known under the name of *strongly perfect* graphs and they were defined by Berge and Duchet in [4]. In [6] and [25] are characterized claw-free graphs that are c-strongly perfect.

Clearly, a graph is c-strongly perfect if and only if its complement is s-strongly perfect, and every c-strongly perfect or s-strongly perfect graph is also perfect (see [4]).

For instance, the graph Y in Figure 1 is not c-strongly perfect, since Y itself has no stable transversal. Hence, its complement $\overline{Y}$ is not s-strongly perfect.

A graph G is *very c-strongly (very s-strongly) perfect* if for each $H \subseteq G$, every vertex of H belongs to a stable (complete, respectively) transversal of H. It is easy to check that P_5 is not very s-strongly perfect, while C_4 is both very s-strongly perfect and very c-strongly perfect.

Unlike the case of perfect graphs, the c-strong perfectness of a graph does not imply the s-strong perfectness of the same graph, and vice-versa. For instance, $K_n, n \geq 1$, is both c-strongly perfect and s-strongly perfect, while $C_{2n}, n \geq 3$, is only c-strongly perfect. In fact, all bipartite graphs are c-strongly perfect.

An interesting case is that of so-called Meyniel graphs (a *Meyniel graph* is one whose each odd cycle of length at least five has at least two chords). Meyniel [11] showed that these graphs are perfect.

Later, Ravindra [20] proved that every Meyniel graph is c-strongly perfect, while Hoang [8] showed that G is a Meyniel graph if and only if it is very c-strongly perfect.

A graph that is both (very) c-strongly perfect and (very) s-strongly perfect is called (very) (c, s)-*strongly perfect* or *co-strongly perfect* (*very co-strongly perfect*, respectively).

Proposition 1. *[4] Every chordal graph is co-strongly perfect and each cograph is c-strongly perfect.*

Since the complement of a cograph is P_4-free, it follows that any cograph is co-strongly perfect.

In fact, in [4] it is proved a stronger result, namely, in a cograph G every maximal stable set meets all the maximal cliques (and this is true also for $\overline{G}$, because $\overline{G}$ is again a cograph).

Now, since each vertex belongs to both a maximal stable set and also to a maximal clique, it follows that each cograph is very co-strongly perfect.

Corollary 1. *The t-perfect graphs and the complete bipartite graphs are very co-strongly perfect.*

There are perfect graphs that are neither s-strongly perfect nor c-strongly perfect; e.g., the graph G from Figure 2, because it has C_6 and $\overline{C_6}$ as induced subgraphs.

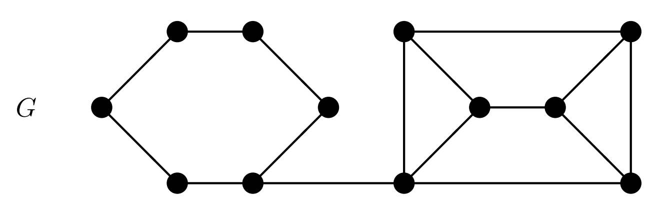

Figure 2: G contains C_6 and $\overline{C_6}$ as induced subgraphs.

If G is not s-strongly (c-strongly) perfect, but $G-v$ is s-strongly (c-strongly, respectively) perfect for each $v \in V(G)$, then G is called *minimally s-strongly (minimally c-strongly, respectively) imperfect.*

For instance, $C_{2n}, n \geq 3$, are perfect and minimally s-strongly imperfect, while every $\overline{C_{2n}}, n \geq 3$, is perfect and minimally s-strongly imperfect.

The imperfect graphs that are minimally strongly imperfect were completely characterized, as follows.

Theorem 2. *[14] A graph G is minimally s-strongly (minimally c-strongly) imperfect and simultaneously imperfect if and only if G or $\overline{G}$ is isomorphic to C_{2k+1} for some $k \geq 2$.*

A list of open problems concerning c-strongly perfect graphs is presented in [21].

Clearly, if G is perfect of some kind, then every induced subgraph of G enjoys the same perfect property.

The *normal product [23]* (also known as the *strong product* [3]) of the graphs $G_i = (V_i, E_i), i = 1, 2$, is the graph $G = G_1 \boxtimes G_2$ having the vertex set

$$V(G) = V_1 \times V_2$$

and the edge set obtained according to the rule: $(x_1, x_2) \sim (y_1, y_2)$ if and only if

$$\begin{array}{rl} \textit{(i)} & x_1 \sim y_1 \text{ and } x_2 = y_2, \text{ or} \\ \textit{(ii)} & x_1 = y_1 \text{ and } x_2 \sim y_2, \text{ or} \\ \textit{(iii)} & x_1 \sim y_1 \text{ and } x_2 \sim y_2. \end{array}$$

For a very detailed treatment of various graph operations (including the normal product, as well), see the book of Imrich and Klavzar, [9].

It is easy to see that:

(i) the normal product is commutative, that is $G_1 \boxtimes G_2$ and $G_2 \boxtimes G_1$ are isomorphic;

(ii) the normal product is associative, i.e., the graphs $(G_1 \boxtimes G_2) \boxtimes G_3$ and $G_1 \boxtimes (G_2 \boxtimes G_3)$ are isomorphic;

(iii) $G_1 \boxtimes G_2$ is connected if and only if both G_1 and G_2 are connected.

Taking into account the above observation *(iii)* and the fact that a graph is:

(a) (very) c-strongly perfect

(b) (very) s-strongly perfect),

(c) t-perfect

if and only if each of its connected components have the same property, we assume in what follows, that all the graphs are connected.

In this paper we discuss co-strong perfectness of normal product of graphs. Strong perfectness of other products of graphs is analysed in [12].

2 Results

The maximal cliques and the transversals of $G_1 \boxtimes G_2$ have a special form, specified in the following.

Proposition 2. (i) *[1] $Q \in \mathcal{C}(G_1 \boxtimes G_2)$ if and only if there exist some $Q_1 \in \mathcal{C}(G_1)$ and $Q_2 \in \mathcal{C}(G_2)$, such that $Q = Q_1 \times Q_2$.*

(ii) *[17] If T_1, T_2 are stable transversals in G_1, G_2, respectively, then $T_1 \times T_2$ is a stable transversal of $G_1 \boxtimes G_2$.*

The perfectness of normal product of graphs is treated by Ravindra in [19], but it is not completely solved yet.

Recall the following known results.

Proposition 3. *[19] If G_1 and G_2 are t-perfect, then $G_1 \boxtimes G_2$ is perfect.*

Later, Alexe and Olaru strengthened this result as follows.

Proposition 4. *[1] If G_1 and G_2 are t-perfect, then $G_1 \boxtimes G_2$ is c-strongly perfect.*

Let us notice that the converse of Theorem 4 is not generally true; e.g., $K_1 \boxtimes C_6$ is c-strongly perfect (since it is isomorphic to C_6), but C_6 is not t-perfect.

The *X-join graph* (see [23]) of the family of graphs $\{G_x : x \in V(X)\}$, indexed by the vertex set $V(X)$ of the graph X, is the graph $G = X[G_x]$ having the vertex set

$$V(G) = \bigcup \{\{x\} \times V(G_x) : x \in V(X)\}$$

and the edge set obtained according to the rule:

$$(x, a) \sim (y, b) \Leftrightarrow \text{either } (i)\ x \sim y \text{ or } (ii)\ x = y \text{ and } a \sim b.$$

If $G_x = Y$ for every $x \in V(X)$, then we write $X[Y]$ and it is known as the *lexicographic product* of X and Y.

It is easy to see that $G \boxtimes K_n$ and $G[K_n]$ are isomorphic.

In [16] (and also, independently, in [24]) it is proved the following result.

Proposition 5. *The graph $X[G_x]$ is (very) c-strongly perfect ((very) s-strongly perfect, co-strongly perfect) if and only if X and every G_x are (very) c-strongly perfect ((very) s-strongly perfect, co-strongly perfect, respectively).*

It is known that if G_1 is a $K_{1,m}$-join of complete graphs and G_2 is a $K_{1,n}$-join of complete graphs, then $G_1 \boxtimes G_2$ is co-strongly perfect [13]. Moreover, the following strengthening is valid.

Proposition 6. *[1] If G_1 is a $K_{1,m}$-join of complete graphs and G_2 is t-perfect, then $G_1 \boxtimes G_2$ is co-strongly perfect.*

As a consequence, we get the following result.

Corollary 2. *If G_1 is a complete graph and G_2 is t-perfect, then $G_1 \boxtimes G_2$ is co-strongly perfect.*

Proof. If $|V(G_1)| = 1$, then $G_1 \boxtimes G_2$ is clearly co-strongly perfect, since it is isomorphic to G_2. If $|V(G_1)| > 1$, then G_1 can be written as a $K_{1,1}$-join of cliques, and according to Proposition 6, it follows immediately that $G_1 \boxtimes G_2$ is co-strongly perfect. □

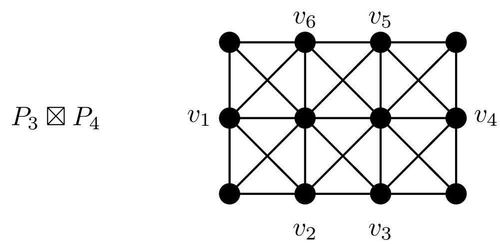

Figure 3: $P_3 \boxtimes P_4$ contains a chordless cycle on 6 vertices : $\{v_i : 1 \leq i \leq 6\}$.

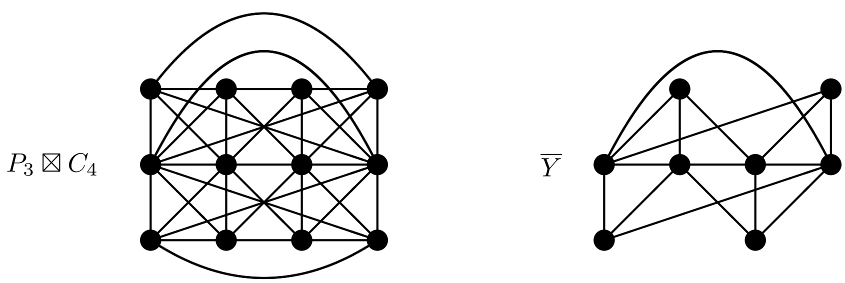

Figure 4: $P_3 \boxtimes C_4$ has $\overline{Y}$ as an induced subgraph.

Let us note that $P_3 \boxtimes P_4$ is not s-strongly perfect, because it contains C_6 as an induced subgraph (see Figure 3).

The graph $P_3 \boxtimes C_4$ is not s-strongly perfect, since it contains $\overline{Y}$ as an induced subgraph (see Figure 4).

Proposition 7. *If none of the graphs G_1 and G_2 is complete, then $G_1 \boxtimes G_2$ is co-strongly perfect if and only if $G_1 \boxtimes G_2$ is s-strongly perfect.*

Proof. If $G_1 \boxtimes G_2$ is co-strongly perfect, then clearly, $G_1 \boxtimes G_2$ is s-strongly perfect.

Conversely, suppose that $G_1 \boxtimes G_2$ is s-strongly perfect.

As it was mentioned above, neither $P_3 \boxtimes P_4$ nor $P_3 \boxtimes C_4$ are s-strongly perfect. On the other hand, if $G_1 \boxtimes G_2$ is s-strongly perfect, then both G_1 and G_2 are s-strongly perfect, because they are isomorphic to some subgraphs of $G_1 \boxtimes G_2$.

If one factor contains P_4 or C_4 as an induced subgraph, then the other factor must be P_3-free, i.e., a complete graph, in contradiction with the hypothesis on G_1 and G_2. Therefore, both factors are (P_4 and C_4)- free, i.e., they are t-perfect. Further, according to Proposition 4, it follows that $G_1 \boxtimes G_2$ is also c-strongly perfect. $\square$

Proposition 8. *Let $G = G_1 \boxtimes G_2$. Then the following assertions are true:*

(i) if G is co-strongly perfect, then either

(a) *one of G_1, G_2 is a complete graph and the other is co-strongly perfect, or*

(b) *both G_1 and G_2 are t-perfect and non-complete;*

(ii) if one of G_1, G_2 is a complete graph and the other is co-strongly perfect, then G is co-strongly perfect.

Proof. *(i)* If G is co-strongly perfect, then both G_1 and G_2 are co-strongly perfect, as being isomorphic to some induced subgraphs of G.

Assume that none of G_1, G_2 is complete, i.e., each has at least one P_3 as an induced subgraph. Since $P_3 \boxtimes P_4$ is not co-strongly perfect (it contains C_6 as an induced subgraph), and $P_3 \boxtimes C_4$ is not co-strongly perfect (it contains $\overline{Y}$ as an induced subgraph), it follows that both G_1 and G_2 must be t-perfect.

(ii) If one of G_1, G_2 is a complete graph, say G_1, and the other is co-strongly perfect, then G is co-strongly perfect, according to Proposition 5, since $G_1 \boxtimes G_2, G_2 \boxtimes G_1$ and $G_2[G_1]$ are pairwise isomorphic. □

Remark 1. (i) *For every $n \geq 4$ the graph $P_3 \boxtimes P_n$ is not s-strongly perfect, as it has C_6 as an induced subgraph.*

(ii) *For every $n \geq 4$ the graph $P_3 \boxtimes C_n$ is not s-strongly perfect, as it has C_{2k} or C_{2k-1}, for $k \geq 3$, as an induced subgraph.*

(iii) *For every $n \geq 5$ the graph $C_4 \boxtimes C_n$ is not s-strongly perfect, as it has C_{2k} or C_{2k-1}, for $k \geq 3$, as an induced subgraph.*

Corollary 3. (i) *$P_m \boxtimes P_n$ is co-strongly perfect if and only if either* (a) *$m \in \{1, 2\}$ and $n \geq 1$, or* (b) *$m = n = 3$.*

(ii) *$P_m \boxtimes C_n$ is co-strongly perfect if and only if either* (a) *$m \in \{1, 2\}$ and $n \in \{3, 4\}$, or* (b) *$m \geq 3$ and $n = 3$.*

(iii) *$C_m \boxtimes C_n$ is co-strongly perfect if and only if $m = 3$ and $n \in \{3, 4\}$.*

Proof. By Proposition 1, we obtain that P_n is co-strongly perfect for all $n \geq 1$.

It is easy to see that C_n is co-strongly perfect only for $n \in \{3, 4\}$.

(i) Assume that $P_m \boxtimes P_n$ is co-strongly perfect. According to Proposition 8*(i)* and Remark 1*(i)*, we infer that either $m \in \{1, 2\}$ and $n \geq 1$, or $m = n = 3$.

Conversely, $P_1 \boxtimes P_n$ and $P_2 \boxtimes P_n$ are chordal graphs for every $n \geq 1$, and hence are co-strongly perfect, by Proposition 1. Proposition 6 ensures that $P_3[K_1] \boxtimes P_3 = K_{1,2}[K_1] \boxtimes P_3$ is co-strongly perfect, since P_3 is t-perfect. Consequently, $P_3 \boxtimes P_3$ is co-strongly perfect, as an induced subgraph of $K_{1,2}[K_1] \boxtimes P_3$.

(ii) Suppose that $P_m \boxtimes C_n$ is co-strongly perfect. In accordance with Proposition 8*(i)* and Remark 1*(ii)*, we get that either $m \in \{1, 2\}$ and $n \in \{3, 4\}$, or $m \geq 3$ and $n = 3$.

The converse follows from Proposition 8*(ii)*.

(iii) Let us note that $C_4 \boxtimes C_4$ is not co-strongly perfect, as having $P_3 \boxtimes C_4$ as an induced subgraph.

If $C_m \boxtimes C_n$ is co-strongly perfect, then Proposition 8*(i)* and Remark 1*(iii)* imply $m = 3$ and $n \in \{3, 4\}$.

The converse follows from Proposition 8*(ii)*. □

As a consequence of Proposition 8, we obtain the following.

Corollary 4. *$G^n = G \boxtimes G \boxtimes G \boxtimes \cdots \boxtimes G, n \geq 2$, is co-strongly perfect if and only if G is a complete graph.*

Proposition 9. *The graph $G_1 \boxtimes G_2$ is very c-strongly perfect (very s-strongly perfect) if and only if one of G_1, G_2 is a complete graph and the other is very c-strongly perfect (very s-strongly perfect, respectively).*

Proof. Let us notice that $P_3 \boxtimes P_3$ is neither very c-strongly perfect (it contains $\overline{P_5}$ as a subgraph) nor very s-strongly perfect (it contains P_5 as a subgraph, see Figure 5). Hence, if $G_1 \boxtimes G_2$ is very c-strongly perfect (very s-strongly perfect, very co-strongly perfect), then both G_1 and G_2 enjoy the same property, and at least one of G_1, G_2 must a complete graph.

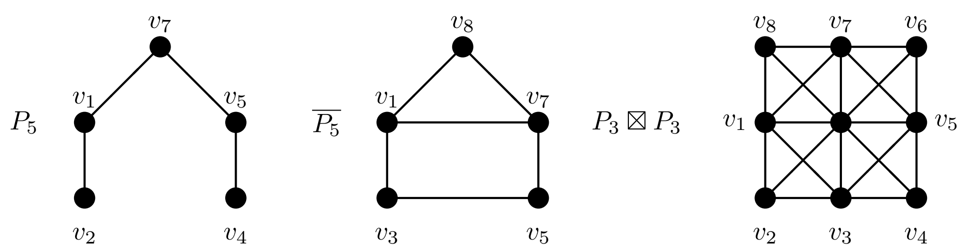

Figure 5: $P_3 \boxtimes P_3$ contains P_5 and $\overline{P_5}$ as induced subgraphs.

The converse follows from Proposition 5, by taking into account that $G \boxtimes K_n$ and $G[K_n]$ are isomorphic. ∎

Corollary 5. *The graph $G_1 \boxtimes G_2 \boxtimes G_3 \boxtimes \cdots \boxtimes G_n$, $n \geq 2$, is very co-strongly perfect if and only if $n-1$ of $G_1, G_2, ..., G_n$ are complete graphs and one is very co-strongly perfect. $G^n = G \boxtimes G \boxtimes G \boxtimes \cdots \boxtimes G, n \geq 2$, is very co-strongly perfect if and only if G is a complete graph.*

Propositions 8 and 4 motivate the following.

Conjecture 1. *If both G_1 and G_2 are non-complete and t-perfect graphs, then $G_1 \boxtimes G_2$ is s-strongly perfect.*

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