Some classifications of Weingarten translation hypersurfaces in Euclidean space
by
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1 Introduction

In the Euclidean space $\mathbb{R}^3$, a surface $M^2$ is called a translation surface if it is given by an immersion

$X : U \subset \mathbb{R}^2 \to \mathbb{R}^3 : (x, y) \mapsto (x, y, f(x) + g(y)),$

where $f(x)$ and $g(y)$ are smooth functions. One famous example of minimal surface in 3-dimensional Euclidean space is the Scherk’s minimal translation surface discovered by Scherk in 1835. In fact, Scherk[10] showed that except that planes, the only minimal translation surfaces are the surfaces given by

$z = \frac{1}{c} \log \left| \frac{\cos cy}{\cos cx} \right|,$

where $c$ is a nonzero constant. This surface, unique up to similarities, is called Scherk’s surface. The concept of translation surfaces was later generalized to hypersurfaces of $\mathbb{R}^{n+1}$ by Dillen, Verstraelen and Zafindratafa [5]. A hypersurface $M \subset \mathbb{R}^{n+1}$ is called a translation hypersurface if it is given by an immersion

$X : \mathbb{R}^n \to \mathbb{R}^{n+1} : (x_1, \cdots, x_n) \mapsto (x_1, \cdots, x_n, f_1(x_1) + \cdots + f_n(x_n)),$

where $f_i$ is a smooth function of one real variable for $i = 1, 2, \cdots, n$. They proved that

**Theorem 1.** Let $M$ be a minimal translation hypersurface in $\mathbb{R}^n$. Then $M$ is either a hyperplane or $M^n = \sum \times \mathbb{R}^{n-2}$, where $\sum$ is a Scherk’s minimal translation surface in $\mathbb{R}^3$.

K. Seo [9] generalized some known results to translation hypersurfaces with constant mean curvature or constant Gauss-Kronecker curvature in Euclidean space $\mathbb{R}^n$ by proving:

**Theorem 2.** Let $M$ be a translation hypersurface with constant mean curvature $H$ in $\mathbb{R}^n$. Then $M$ is congruent to a cylinder $\sum \times \mathbb{R}^{n-2}$, where $\sum$ is a constant mean curvature surface in $\mathbb{R}^3$. In particular, if $H = 0$, then $M$ is either a hyperplane or $\sum \times \mathbb{R}^{n-2}$, where $\sum$ is a Scherk’s minimal translation surface in $\mathbb{R}^3$.

**Theorem 3.** Let $M$ be a translation hypersurface with constant Gauss-Kronecker curvature $GK$ in $\mathbb{R}^n$. Then $M$ is congruent to a cylinder, and hence $GK = 0$. 
Recently, Lima-Santos-Sousa [8] obtained the complete classification of translation hypersurfaces of $\mathbb{R}^{n+1}$ with zero scalar curvature.

**Theorem 4.** Let $M^n$ $(n \geq 3)$ be a translation hypersurface in $\mathbb{R}^{n+1}$. Then $M^n$ has zero scalar curvature, if and only if, it is congruent to the graph of a vertical cylinder or a generalized periodic Enneper hypersurface.

Furthermore, the authors [8] proved that any translation hypersurfaces in $\mathbb{R}^{n+1}$ with constant scalar curvature must have zero scalar curvature.

On the other hand, many geometers have approached the problem of characterizing hypersurfaces with constant mean curvature or with constant scalar curvature in real space forms and obtained some classical results. Furthermore, when the scalar curvature is proportional to the mean curvature, that is $R = kH$, where $k$ is constant, Li [7] characterized the rigidity of compact hypersurfaces with nonnegative sectional curvature. Concerning this kind of hypersurfaces, there are some other results, such as [4, 12]. Next, Li et al. [6] extended the result of [7] by considering linear Weingarten hypersurfaces immersed in the unit sphere, that is, hypersurfaces whose mean curvature $H$ and normalized scalar curvature $R$ satisfy $R = aH + b$, where $a$ and $b$ are constants. Thereafter, there are lots of rigidity and the characterization results [11, 1, 3, 2] of linear Weingarten hypersurfaces in real space forms, which generalized the classical constant scalar curvature hypersurfaces and the hypersurfaces with $R = kH$.

Based on above results, it is necessary and interesting to give the complete classification of translation Weingarten hypersurfaces. Of course, the problem is difficult without any other geometric condition. In this paper, we firstly consider the translation Weingarten hypersurface in 4-dimensional Euclidean space $\mathbb{R}^4$ with $R = kH$. Precisely, we get the following results.

**Theorem 5.** Let $M$ be a translation Weingarten hypersurface in $\mathbb{R}^4$ with $R = kH$. Then $M$ is congruent to one of the following surfaces:

1. A cylinder
   \[
   X(x_1, x_2, x_3) = (x_1, x_2, x_3, a_1 x_1 + a_2 x_2 + f_3(x_3) + c),
   \]
   where $a_1, a_2, c$ are constant;

2. A periodic Enneper hypersurface
   \[
   X(x_1, x_2, x_3) = (x_1, x_2, x_3, a_1 \ln \left| \frac{\cos \left( -\frac{x_1}{a_1 + a_2} + b_3 \right)}{\cos \left( \frac{x_1}{a_1} + b_1 \right)} \right| + a_2 \ln \left| \frac{\cos \left( -\frac{x_1}{a_1 + a_2} + b_3 \right)}{\cos \left( \frac{x_2}{a_2} + b_2 \right)} \right| + c),
   \]
   where $a_1, a_2, b_1, b_2, b_3, c$ are constants.

Besides, we will consider the translation Weingarten hypersurfaces with $GK = kH$ and give a complete classification in $n$-dimensional Euclidean space in $\mathbb{R}^n$. Precisely, we obtain:

**Theorem 6.** Let $M$ be a translation hypersurface in $\mathbb{R}^n$ with $GK = kH$. Then $M$ is congruent to a cylinder, and hence $GK = 0$. 

\[
\text{Weingarten translation hypersurfaces}
\]
2 Preliminaries

Let $M^n$ be a translation hypersurface immersed in $\mathbb{R}^{n+1}$ given by

$$X : \mathbb{R}^n \to \mathbb{R}^{n+1} : (x_1, \ldots, x_n) \mapsto (x_1, \ldots, x_n, f_1(x_1) + \cdots + f_n(x_n)),$$

and each $f_i$ is a smooth function for $i = 1, \cdots, n$. One can easily see that the unit normal vector $N$ is given by

$$N = \left( -f'_1, \ldots, -f'_n, 1 \right) W,$$

where

$$W = \sqrt{1 + f''_1^2 + \cdots + f''_n^2}.$$

It is easy to check that the coefficient $g_{ij} = g \left( \frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j} \right)$ of the metric tensor and the inverse matrix $(g^{ij})$ of $(g_{ij})$ is given by

$$g_{ij} = \delta_{ij} + f'_i f'_j, \quad g^{ij} = \delta_{ij} - f'_i f'_j W^2.$$

The matrix of the second fundamental form $h$ is

$$h_{ii} = \frac{f''_i}{W}, \quad h_{ij} = 0 \quad (i \neq j),$$

where $f''_i = \frac{d^2 f_i}{dx_i^2}$. Then the matrix $A = (a^i_j)$ of the shape operator is given by

$$a^i_j = \sum_k h_{ik} g^{kj} = \frac{f''_i}{W} - \frac{f''_i f''_j}{W^3}.$$

Since $nH = \sum_i a^i_i$, then the mean curvature $H$ is given by

$$H = \frac{1}{nW^3} \sum_i \left( 1 + \sum_{j \neq i} f''_j \right) f''_i. \quad (2.1)$$

It is easy to check that the Gauss-Kronecker curvature $GK$ is

$$GK = \frac{\det h_{ij}}{\det g_{ij}} = \frac{f''_1 f''_2 \cdots f''_n}{W^{n+2}}. \quad (2.2)$$

We denote $\lambda_1, \ldots, \lambda_n$ the principle curvatures of hypersurface $M^n$ and $S_r$ be $r$-th elementary symmetric polynomials given by $S_r = \sum \lambda_1 \cdots \lambda_r$, where $r = 1, \cdots, n$ and $1 \leq i_1 < \cdots < i_r \leq n$. In particular, $S_1 = nH$, $S_n = GK$ and $S_2$ relates to the scalar curvature $R$ in the following form:

$$n(n-1)R = 2S_2. \quad (2.3)$$

Next we recall a result in [8] for later use.

**Proposition 1.** Let $M^n$ be a translation hypersurface immersed in $\mathbb{R}^{n+1}$ parameterized by

$$X : \mathbb{R}^n \to \mathbb{R}^{n+1} : (x_1, \ldots, x_n) \mapsto (x_1, \ldots, x_n, f_1(x_1) + \cdots + f_n(x_n)),$$

where $f_i$ is a smooth function of one real variable for $i = 1, \cdots, n$. Then

$$S_2 = \frac{1}{W^4} \sum_{1 \leq i < j \leq n} f''_i f''_j \left( 1 + \sum_{1 \leq k \leq n; k \neq i, j} f''_k \right). \quad (2.4)$$

3 Proof of the theorems

Proof of Theorem 5. Let $M^3$ be a translation hypersurface in $\mathbb{R}^4$. It follows from (2.1) and (2.4) that

$$H = \frac{\sum a_i^i}{3} = \frac{(1 + f_2^2 + f_3^2)f_1'' + (1 + f_1^2 + f_3^2)f_2'' + (1 + f_1^2 + f_3^2)f_3''}{3W^3}, \quad (3.1)$$

and

$$S_2 = \frac{f_1''f_2''(1 + f_3^2) + f_1''f_3''(1 + f_2^2) + f_2''f_3''(1 + f_1^2)}{W^4}. \quad (3.2)$$

It follows from (2.3) that the condition $R = kH$ is equivalent to $S_2 = 3kH$. So from (3.1) and (3.2), we have

$$f_1''f_2''(1 + f_3^2) + f_1''f_3''(1 + f_2^2) + f_2''f_3''(1 + f_1^2) = kW[(1 + f_2^2 + f_3^2)f_1'' + (1 + f_1^2 + f_3^2)f_2'' + (1 + f_1^2 + f_2^2)f_3'']. \quad (3.3)$$

In order to give the proof of Theorem 5, we will consider two cases as following:

Case A. $f_1''f_2''f_3'' \neq 0$. In this case, divided by $f_1''f_2''f_3''$ on both sides of (3.3), we have

$$\frac{1 + f_1^2}{f_1''} + \frac{1 + f_2^2}{f_2''} + \frac{1 + f_3^2}{f_3''} = kW(\frac{1 + f_2^2 + f_3^2}{f_1''f_2''} + \frac{1 + f_1^2 + f_3^2}{f_1''f_3''} + \frac{1 + f_1^2 + f_2^2}{f_2''f_3''}). \quad (3.4)$$

Case A.1. $k = 0$. In this case, (3.4) implies that there exist constants $a_1, a_2, a_3$ such that

$$\frac{1 + f_1^2}{f_1''} = a_1, \quad \frac{1 + f_2^2}{f_2''} = a_2, \quad \frac{1 + f_3^2}{f_3''} = a_3, \quad (3.5)$$

and $a_1 + a_2 + a_3 = 0$. Solving the system (3.5), we have

$$f_1 = -a_1 \ln |\cos(\frac{x_1}{a_1} + b_1)| + c_1,$$

$$f_2 = -a_2 \ln |\cos(\frac{x_2}{a_2} + b_2)| + c_2,$$

$$f_3 = -a_3 \ln |\cos(\frac{x_3}{a_3} + b_3)| + c_3 = (a_1 + a_2) \ln |\cos(\frac{x_1}{-(a_1 + a_2)} + b_3)| + c_3,$$  

where $b_1, b_2, b_3$ and $c_1, c_2, c_3$ are constants. Thus we get case (2) in Theorem 5.

Case A.2. $k \neq 0$. Differentiate the equation (3.4) with respect to $x_1$, we have

$$\frac{(1 + f_1^2)^2}{f_1''} = kW(\frac{1 + f_2^2 + f_3^2}{f_1''f_2''} + \frac{1 + f_1^2 + f_3^2}{f_1''f_3''} + \frac{1 + f_1^2 + f_2^2}{f_2''f_3''}) x_1.$$

$$+ kW(\frac{1 + f_2^2 + f_3^2}{f_1''f_2''} + \frac{1 + f_1^2 + f_3^2}{f_1''f_3''} + \frac{1 + f_1^2 + f_2^2}{f_2''f_3''}) (x_1). \quad (3.7)$$
By (3.4), equation (3.7) becomes
\[
\left( \frac{1 + f_1^2}{f_1'} \right)' = \frac{f_1 f_1''}{W^2} \left[ \frac{1 + f_1^2}{f_1'} + \frac{1 + f_2^2}{f_2'} + \frac{1 + f_3^2}{f_3'} \right] \\
+ kW \left[ \left( \frac{1 + f_1^2 + f_2^2}{f_1' f_2'} \right)_{x_1} + \left( \frac{1 + f_1^2 + f_3^2}{f_1' f_3'} \right)_{x_1} \right].
\] (3.8)

Differentiating the equation (3.8) with respect to \( x_2 \), we have
\[
0 = -\frac{2 f_1 f_1'' f_2 f_2''}{W^4} \left[ \frac{1 + f_1^2}{f_1'} + \frac{1 + f_2^2}{f_2'} + \frac{1 + f_3^2}{f_3'} \right] + \frac{f_1 f_1''}{W^2} \left( \frac{1 + f_2^2}{f_2'} \right)'
+ kW \left[ \left( \frac{1 + f_1^2 + f_2^2}{f_1' f_2'} \right)_{x_1} + \left( \frac{1 + f_1^2 + f_3^2}{f_1' f_3'} \right)_{x_1} \right] + kW \left( \frac{1 + f_1^2 + f_2^2}{f_1' f_2'} \right)_{x_1 x_2}.
\] (3.9)

By (3.8), equation (3.9) becomes
\[
0 = -\frac{2 f_1 f_1'' f_2 f_2''}{W^4} \left[ \frac{1 + f_1^2}{f_1'} + \frac{1 + f_2^2}{f_2'} + \frac{1 + f_3^2}{f_3'} \right] + \frac{f_1 f_1''}{W^2} \left( \frac{1 + f_2^2}{f_2'} \right)'
+ kW \left( \frac{1 + f_1^2 + f_2^2}{f_1' f_2'} \right)_{x_1 x_2},
\] (3.10)

that is
\[
\frac{3 f_1 f_1'' f_2 f_2''}{W^5} \left[ \frac{1 + f_1^2}{f_1'} + \frac{1 + f_2^2}{f_2'} + \frac{1 + f_3^2}{f_3'} \right] = \frac{f_1 f_1''}{W^3} \left( \frac{1 + f_2^2}{f_1'} \right)' + f_1 f_1'' \left( \frac{1 + f_2^2}{f_2'} \right)' + kW \left( \frac{1 + f_1^2 + f_2^2}{f_1' f_2'} \right)_{x_1 x_2}. \] (3.11)

Differentiating the equation (3.11) with respect to \( x_3 \), we have
\[
\frac{5 f_1 f_1'' f_2 f_2'' f_3 f_3''}{W^2} \left[ \frac{1 + f_1^2}{f_1'} + \frac{1 + f_2^2}{f_2'} + \frac{1 + f_3^2}{f_3'} \right] = f_2 f_2'' f_2'' f_3 f_3'' \left( \frac{1 + f_1^2}{f_1'} \right)' + f_1 f_1'' f_2 f_2'' \left( \frac{1 + f_2^2}{f_2'} \right)' + f_1 f_1'' f_3 f_3'' \left( \frac{1 + f_3^2}{f_3'} \right)'.
\] (3.12)

Divided by \( f_1 f_1'' f_2 f_2'' f_3 f_3'' \) on both sides of (3.12), we have
\[
\frac{5}{W^2} \left[ \frac{1 + f_1^2}{f_1'} + \frac{1 + f_2^2}{f_2'} + \frac{1 + f_3^2}{f_3'} \right] = \frac{1}{f_1 f_1''} \left( \frac{1 + f_1^2}{f_1'} \right)' + \frac{1}{f_2 f_2''} \left( \frac{1 + f_2^2}{f_2'} \right)' + \frac{1}{f_3 f_3''} \left( \frac{1 + f_3^2}{f_3'} \right)'.
\] (3.13)
Next we denote
\[ F = \frac{1 + f_1'^2}{f_1''}, \quad G = \frac{1 + f_2'^2}{f_2''}, \quad M = \frac{1 + f_3'^2}{f_3''}. \] (3.14)

Then (3.13) becomes
\[ \frac{5}{W^2} [F + G + M] = \frac{F'}{f_1''} + \frac{G'}{f_2''} + \frac{M'}{f_3''}. \] (3.15)

Differentiating the equation (3.15) with respect to \( x_1 \), we have
\[ -\frac{10 f_1' f_1''}{W^4} [F + G + M] + \frac{5}{W^2} F' = \left( \frac{F'}{f_1''} \right)' . \] (3.16)

Divided by \( 5 f_1' f_1'' \) on both sides of equation (3.16), we have
\[ -\frac{2}{W^4} [F + G + M] + \frac{1}{W^2} \frac{F'}{f_1''} = \frac{1}{5 f_1' f_1''} \left( \frac{F'}{f_1''} \right)' . \] (3.17)

Differentiating the equation (3.17) with respect to \( x_2 \), we have
\[ \frac{4 f_2' f_2''}{W^6} [F + G + M] - \frac{1}{W^4} G' - \frac{f_2' f_2''}{W^4} \frac{F'}{f_1''} = 0. \] (3.18)

Divided by \( f_2' f_2'' \) on both sides of equation (3.18), we have
\[ \frac{4}{W^2} [F + G + M] - \frac{G'}{f_2''} - \frac{F'}{f_1''} = 0. \] (3.19)

Differentiating the equation (3.19) with respect to \( x_3 \) and divided by \( f_3' f_3'' \), we have
\[ \frac{M'}{f_3''} = \frac{2}{W^2} (F + G + M) . \] (3.20)

Since equation (3.15) is symmetric with \( F, G \) and \( M \), so we have
\[ \frac{F'}{f_1''} = \frac{G'}{f_2''} = \frac{2}{W^2} (F + G + M) . \] (3.21)

Therefore
\[ \frac{F'}{f_1''} + \frac{G'}{f_2''} + \frac{M'}{2 f_3''} = \frac{6}{W^2} (F + G + M) , \] (3.22)

which together with (3.15) gives \( F + G + M = 0 \) and so \( F, G, M \) are constants. Similar to case A.1, we also get case (2) in Theorem 5.

Case B. \( f_1'' f_2'' f_3'' = 0 \). In this case, at least one of \( f_1'', f_2'' \) and \( f_3'' \) vanishes. Without loss of generality, we assume \( f_1'' = 0 \) and hence there exists constant \( c \) such that \( f_1' = c \), then equation (3.3) becomes
\[ \frac{f_2'' f_3'}{(1 + c^2)} = kW \left[ (1 + c^2 + f_3'^2) f_2'' + (1 + c^2 + f_2'^2) f_3'' \right] . \] (3.23)
It follows from (2.1) and (2.2) that

\begin{align}
\text{Theorem 5.}
\end{align}

which implies that

\[ f = 0. \]

If one of \( f'_2 \) and \( f''_3 \) is zero, we get case (1) in Theorem 5.

**Case B.1.** \( k = 0 \). From (3.23), we have \( f''_2 f'''_3 = 0 \). If \( f''_2 = f'''_3 = 0 \), then the hypersurface \( M \) is a hyperplane in \( \mathbb{R}^4 \) and we get case (1) in Theorem 5 with \( f_3 = ax_3 + b \). If one of \( f''_2 \) and \( f'''_3 \) is zero, we get case (1) in Theorem 5.

**Case B.2.** \( k \neq 0 \). If \( f''_2 = 0 \), from (3.23) we have \( f'''_3 = 0 \). Then there exist constants \( a_i, b_i, c_i (1 \leq i \leq 3) \) such that \( f_1 = a_1 x_1 + a_2 x_2 + b_1 x_1 + b_2 x_2 + c_1 x_1 + c_2 \), and the hypersurface \( M \) is a hyperplane in \( \mathbb{R}^4 \). We get the case (1) in Theorem 5.

If \( f'''_3 \neq 0 \), from (3.23) we have \( f''_2 \neq 0 \). Divided by \( f''_2 f'''_3 \) on both sides of equation (3.23), we have

\[
(1 + c^2) = kW \left[ \frac{1 + c^2 + f''_3}{f''_3} + \frac{1 + c^2 + f''_2}{f''_2} \right].
\] (3.24)

Differentiating the equation (3.24) with respect to \( x_2 \), we have

\[
k \frac{f''_3 f''_2}{W^2} \left[ \frac{1 + c^2 + f''_3}{f''_3} + \frac{1 + c^2 + f''_2}{f''_2} \right] = -k \left( \frac{1 + c^2 + f''_2}{f''_2} \right)'.
\] (3.25)

Since \( k \neq 0 \), then by (3.24), equation (3.25) becomes

\[
f''_3 f''_2 \frac{1 + c^2}{W^2} = -k \left( \frac{1 + c^2 + f''_2}{f''_2} \right)'.
\] (3.26)

Differentiating the equation (3.26) with respect to \( x_3 \), we have

\[
\frac{1}{W^3} f''_2 f'''_2 f''_3 f'''_3 \left[ 1 + c^2 \right] = 0,
\] (3.27)

which implies that \( f''_2 f'''_3 = 0 \) and this is a contradiction. This completes the proof of Theorem 5.

**Proof of Theorem 6.** Let \( M^n \) be a Weingarten hypersurface with \( GK = kH \) in \( \mathbb{R}^{n+1} \). It follows from (2.1) and (2.2) that

\[
\frac{f''_2 f'''_2 \cdots f''_n}{(1 + \sum_{i=1}^{n} f''_i)^{(n+2)/2}} = k \frac{\sum_{i=1}^{n} (1 + \sum_{j \neq i} f''_j) f''_i}{n(1 + \sum_{i=1}^{n} f''_i)^{n/2}},
\]

that is

\[
\frac{f''_2 f'''_2 \cdots f''_n}{(1 + \sum_{i=1}^{n} f''_i)^{(n+1)/2}} = k \frac{n \sum_{i=1}^{n} (1 + \sum_{j \neq i} f''_j) f''_i}{n(1 + \sum_{i=1}^{n} f''_i)^{n/2}}.
\] (3.28)

Assume that \( f''_2 f'''_2 \cdots f''_n \neq 0 \), dividing (3.28) by \( f''_2 f'''_2 \cdots f''_n \), we have

\[
\frac{1}{(1 + \sum_{i=1}^{n} f''_i)^{(n-1)/2}} = k \frac{n \sum_{i=1}^{n} 1 + \sum_{j \neq i} f''_j}{\prod_{j \neq i}^{n} f''_j}.
\] (3.29)

Differentiating the equation (3.29) with respect to \( x_1, x_2, \ldots, x_n \) successively. We have

\[
(-1)^n n \prod_{i=1}^{n} (n + 2i - 3)(1 + f''_1^2 + \cdots + f''_n^2) - 2n-1 f''_1 f''_2 \cdots f''_n f''_n = 0.
\] (3.30)
This implies that \( f_1' f_2' \cdots f_n' = 0 \), which contradicts to the assumption \( f_1'' f_2'' \cdots f_n'' \neq 0 \).

Since \( f_1' f_2' \cdots f_n' = 0 \), there exists at least one \( f_i' = 0 \), we assume that \( f_1'' = 0 \) and hence \( f_1 = ax_1 + b \), where \( a \) and \( b \) are constants. It follows immediately that

\[
X(x_1, \cdots, x_n) = (x_1, \cdots, x_n, f_1(x_1) + f_2(x_2) + \cdots + f_n(x_n))
\]

\[
= (x_1, \cdots, x_n, ax_1 + b + f_2(x_2) + \cdots + f_n(x_n))
\]

\[
= x_1(1, 0, \cdots, 0, a) + (0, x_2, \cdots, x_n, b + f_2(x_2) + \cdots + f_n(x_n)),
\]

which implies that \( M^n \) is a cylinder. This completes the proof of Theorem 6.

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