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Some classifications of Weingarten translation hypersurfaces in Euclidean space

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1 Introduction

In the Euclidean space $\mathbb{R}^3$, a surface M^2 is called a translation surface if it is given by an immersion

$$X : U \subset \mathbb{R}^2 \to \mathbb{R}^3 : (x, y) \mapsto \big(x, y, f(x) + g(y)\big),$$

where $f(x)$ and $g(y)$ are smooth functions. One famous example of minimal surface in 3-dimensional Euclidean space is the Scherk's minimal translation surface discovered by Scherk in 1835. In fact, Scherk[10] showed that except that planes, the only minimal translation surfaces are the surfaces given by

$$z = \frac{1}{c} \log \Big| \frac{\cos cy}{\cos cx} \Big|,$$

where c is a nonzero constant. This surface, unique up to similarities, is called Scherk's surface. The concept of translation surfaces was later generalized to hypersurfaces of $\mathbb{R}^{n+1}$ by Dillen, Verstraelen and Zafindratafa [5]. A hypersurface $M \subset \mathbb{R}^{n+1}$ is called a translation hypersurface if it is given by an immersion

$$X : \mathbb{R}^n \to \mathbb{R}^{n+1} : (x_1, \cdots, x_n) \mapsto \big(x_1, \cdots, x_n, f_1(x_1) + \cdots + f_n(x_n)\big),$$

where f_i is a smooth function of one real variable for $i = 1, 2, \cdots, n$. They proved that

Theorem 1. *Let M be a minimal translation hypersurface in $\mathbb{R}^n$. Then M is either a hyperplane or $M^n = \sum \times \mathbb{R}^{n-2}$, where $\sum$ is a Scherk's minimal translation surface in $\mathbb{R}^3$.*

K. Seo [9] generalized some known results to translation hypersurfaces with constant mean curvature or constant Gauss-Kronecker curvature in Euclidean space $\mathbb{R}^n$ by proving:

Theorem 2. *Let M be a translation hypersurface with constant mean curvature H in $\mathbb{R}^n$. Then M is congruent to a cylinder $\sum \times \mathbb{R}^{n-2}$, where $\sum$ is a constant mean curvature surface in $\mathbb{R}^3$. In particular, if $H = 0$, then M is either a hyperplane or $\sum \times \mathbb{R}^{n-2}$, where $\sum$ is a Scherk's minimal translation surface in $\mathbb{R}^3$.*

Theorem 3. *Let M be a translation hypersurface with constant Gauss-Kronecker curvature GK in $\mathbb{R}^n$. Then M is congruent to a cylinder, and hence $GK = 0$.*

Recently, Lima-Santos-Sousa[8] obtained the complete classification of translation hypersurfaces of $\mathbb{R}^{n+1}$ with zero scalar curvature.

Theorem 4. *Let M^n $(n \geq 3)$ be a translation hypersurface in $\mathbb{R}^{n+1}$. Then M^n has zero scalar curvature, if and only if, it is congruent to the graph of a vertical cylinder or a generalized periodic Enneper hypersurface.*

Furthermore, the authors [8] proved that any translation hypersurfaces in $\mathbb{R}^{n+1}$ with constant scalar curvature must have zero scalar curvature.

On the other hand, many geometers have approached the problem of characterizing hypersurfaces with constant mean curvature or with constant scalar curvature in real space forms and obtained some classical results. Furthermore, when the scalar curvature is proportional to the mean curvature, that is $R = kH$, where k is constant, Li [7] characterized the rigidity of compact hypersurfaces with nonnegative sectional curvature. Concerning this kind of hypersurfaces, there are some other results, such as [4, 12]. Next, Li et al. [6] extended the result of [7] by considering linear Weingarten hypersurfaces immersed in the unit sphere, that is, hypersurfaces whose mean curvature H and normalized scalar curvature R satisfy $R = aH + b$, where a and b are constants. Thereafter, there are lots of rigidity and the characterization results [11, 1, 3, 2] of linear Weingarten hypersurfaces in real space forms, which generalized the classical constant scalar curvature hypersurfaces and the hypersurfaces with $R = kH$.

Based on above results, it is necessary and interesting to give the complete classification of translation weingarten hypersurfaces. Of course, the problem is difficult without any other geometric condition. In this paper, we firstly consider the translation Weingarten hypersurface in 4-dimensional Euclidean space $\mathbb{R}^4$ with $R = kH$. Precisely, we get the following results.

Theorem 5. *Let M be a translation weingarten hypersurface in $\mathbb{R}^4$ with $R = kH$. Then M is congruent to one of the following surfaces:*

1. *A cylinder*
$$X(x_1, x_2, x_3) = \big(x_1, x_2, x_3, a_1x_1 + a_2x_2 + f_3(x_3) + c\big),$$
where a_1, a_2, c are constant;

2. *A periodic Enneper hypersurface*
$$X(x_1, x_2, x_3) = \Big(x_1, x_2, x_3, a_1 \ln\Big|\frac{\cos(-\frac{x_1}{a_1+a_2} + b_3)}{\cos(\frac{x_1}{a_1} + b_1)}\Big| + a_2 \ln\Big|\frac{\cos(-\frac{x_1}{a_1+a_2} + b_3)}{\cos(\frac{x_2}{a_2} + b_2)}\Big| + c\Big),$$
where $a_1, a_2, b_1, b_2, b_3, c$ are constants.

Besides, we will consider the translation Weingarten hypersurfaces with $GK = kH$ and give a complete classification in n-dimensional Euclidean space in $\mathbb{R}^n$. Precisely, we obtain:

Theorem 6. *Let M be a translation hypersurface in $\mathbb{R}^n$ with $GK = kH$. Then M is congruent to a cylinder, and hence $GK = 0$.*

2 Preliminaries

Let M^n be a translation hypersurface immersed in $\mathbb{R}^{n+1}$ given by

$$X:\mathbb{R}^n\to\mathbb{R}^{n+1}:\big(x_1,\cdots,x_n\big)\mapsto\big(x_1,\cdots,x_n,f_1(x_1)+\cdots+f_n(x_n)\big),$$

and each f_i is a smooth function for $i=1,\cdots,n$. One can easily see that the unit normal vector N is given by

$$N=\frac{(-f_1',\cdots,-f_n',1)}{W},$$

where

$$W=\sqrt{1+f_1'^2+\cdots+f_n'^2}.$$

It is easy to check that the coefficient $g_{ij}=g(\frac{\partial}{\partial x_i},\frac{\partial}{\partial x_j})$ of the metric tensor and the inverse matrix (g^{ij}) of (g_{ij}) is given by

$$g_{ij}=\delta_{ij}+f_i'f_j',\qquad g^{ij}=\delta_{ij}-\frac{f_i'f_j'}{W^2}.$$

The matrix of the second fundamental form h is

$$h_{ii}=\frac{f_i''}{W},\qquad h_{ij}=0\quad(i\neq j),$$

where $f_i''=\frac{d^2f}{dx_i^2}$. Then the matrix $A=(a_i^j)$ of the shape operator is given by

$$a_i^j=\sum_k h_{ik}g^{kj}=\frac{f_i''}{W}-\frac{f_i''f_i'f_j'}{W^3}.$$

Since $nH=\sum_i a_i^i$, then the mean curvature H is given by

$$H=\frac{1}{nW^3}\sum_i\big(1+\sum_{j\neq i}f_j'^2\big)f_i''. \tag{2.1}$$

It is easy to check that the Gauss-Kronecker curvature GK is

$$GK=\frac{\det h_{ij}}{\det g_{ij}}=\frac{f_1''f_2''\cdots f_n''}{W^{n+2}}. \tag{2.2}$$

We denote $\lambda_1,\cdots,\lambda_n$ the principle curvatures of hypersurface M^n and S_r be r-th elementary symmetric polynomials given by $S_r=\sum\lambda_{i_1}\cdots\lambda_{i_r}$, where $r=1,\cdots,n$ and $1\leq i_1<\cdots<i_r\leq n$. In particular, $S_1=nH$, $S_n=GK$ and S_2 relates to the scalar curvature R in the following form:

$$n(n-1)R=2S_2. \tag{2.3}$$

Next we recall a result in [8] for later use.

Proposition 1. *Let M^n be a translation hypersurface immersed in $\mathbb{R}^{n+1}$ parameterized by*

$$X:\mathbb{R}^n\to\mathbb{R}^{n+1}:\big(x_1,\cdots,x_n\big)\mapsto\big(x_1,\cdots,x_n,f_1(x_1)+\cdots+f_n(x_n)\big),$$

where f_i is a smooth function of one real variable for $i=1,\cdots,n$. Then

$$S_2=\frac{1}{W^4}\sum_{1\leq i<j\leq n}f_i''f_j''(1+\sum_{1\leq k\leq n;k\neq i,j}f_k'^2). \tag{2.4}$$

3 Proof of the theorems

Proof of Theorem 5. Let M^3 be a translation hypersurface in $\mathbb{R}^4$. It follows from (2.1) and (2.4) that

$$H = \frac{\sum_i a_i^i}{3} = \frac{(1+f_2'^2+f_3'^2)f_1'' + (1+f_1'^2+f_3'^2)f_2'' + (1+f_1'^2+f_2'^2)f_3''}{3W^3}, \tag{3.1}$$

and

$$S_2 = \frac{f_1''f_2''(1+f_3'^2) + f_1''f_3''(1+f_2'^2) + f_2''f_3''(1+f_1'^2)}{W^4}. \tag{3.2}$$

It follows from (2.3) that the condition $R = kH$ is equivalent to $S_2 = 3kH$. So from (3.1) and (3.2), we have

$$\begin{aligned} &f_1''f_2''(1+f_3'^2) + f_1''f_3''(1+f_2'^2) + f_2''f_3''(1+f_1'^2) \\ &= kW\big[(1+f_2'^2+f_3'^2)f_1'' + (1+f_1'^2+f_3'^2)f_2'' + (1+f_1'^2+f_2'^2)f_3''\big]. \end{aligned} \tag{3.3}$$

In order to give the proof of Theorem 5, we will consider two cases as following:

Case A. $f_1''f_2''f_3'' \neq 0$. In this case, divided by $f_1''f_2''f_3''$ on both sides of (3.3), we have

$$\frac{1+f_1'^2}{f_1''} + \frac{1+f_2'^2}{f_2''} + \frac{1+f_3'^2}{f_3''} = kW\Big(\frac{1+f_1'^2+f_2'^2}{f_1''f_2''} + \frac{1+f_1'^2+f_3'^2}{f_1''f_3''} + \frac{1+f_2'^2+f_3'^2}{f_2''f_3''}\Big). \tag{3.4}$$

Case A.1. $k = 0$. In this case, (3.4) implies that there exist constants a_1, a_2, a_3 such that

$$\frac{1+f_1'^2}{f_1''} = a_1, \quad \frac{1+f_2'^2}{f_2''} = a_2, \quad \frac{1+f_3'^2}{f_3''} = a_3, \tag{3.5}$$

and $a_1 + a_2 + a_3 = 0$. Solving the system (3.5), we have

$$\begin{aligned} f_1 &= -a_1 \ln|\cos(\frac{x_1}{a_1} + b_1)| + c_1, \\ f_2 &= -a_2 \ln|\cos(\frac{x_2}{a_2} + b_2)| + c_2, \\ f_3 &= -a_3 \ln|\cos(\frac{x_3}{a_3} + b_3)| + c_3 = (a_1+a_2)\ln|\cos(\frac{x_1}{-(a_1+a_2)} + b_3)| + c_3, \end{aligned} \tag{3.6}$$

where b_1, b_2, b_3 and c_1, c_2, c_3 are constants. Thus we get case (2) in Theorem 5.

Case A.2. $k \neq 0$. Differentiate the equation (3.4) with respect to x_1, we have

$$\begin{aligned} (\frac{1+f_1'^2}{f_1''})' \;&=\; k\frac{f_1'f_1''}{W}\Big[\frac{1+f_1'^2+f_2'^2}{f_1''f_2''} + \frac{1+f_1'^2+f_3'^2}{f_1''f_3''} + \frac{1+f_2'^2+f_3'^2}{f_2''f_3''}\Big] \\ &+\; kW\Big[(\frac{1+f_1'^2+f_2'^2}{f_1''f_2''})_{x_1} + (\frac{1+f_1'^2+f_3'^2}{f_1''f_3''})_{x_1}\Big]. \end{aligned} \tag{3.7}$$

By (3.4), equation (3.7) becomes

$$\begin{aligned}(\frac{1+f_1^{'2}}{f_1''})' &= \frac{f_1'f_1''}{W^2}\Big[\frac{1+f_1^{'2}}{f_1''}+\frac{1+f_2^{'2}}{f_2''}+\frac{1+f_3^{'2}}{f_3''}\Big]\\ &+ kW\Big[(\frac{1+f_1^{'2}+f_2^{'2}}{f_1''f_2''})_{x_1}+(\frac{1+f_1^{'2}+f_3^{'2}}{f_1''f_3''})_{x_1}\Big].\end{aligned} \tag{3.8}$$

Differentiating the equation (3.8) with respect to x_2, we have

$$\begin{aligned}0 &= -\frac{2f_1'f_1''f_2'f_2''}{W^4}\Big[\frac{1+f_1^{'2}}{f_1''}+\frac{1+f_2^{'2}}{f_2''}+\frac{1+f_3^{'2}}{f_3''}\Big]+\frac{f_1'f_1''}{W^2}(\frac{1+f_2^{'2}}{f_2''})'\\ &+ \frac{kf_2'f_2''}{W}\Big[(\frac{1+f_1^{'2}+f_2^{'2}}{f_1''f_2''})_{x_1}+(\frac{1+f_1^{'2}+f_3^{'2}}{f_1''f_3''})_{x_1}\Big]+kW(\frac{1+f_1^{'2}+f_2^{'2}}{f_1''f_2''})_{x_1x_2}.\end{aligned} \tag{3.9}$$

By (3.8), equation (3.9) becomes

$$\begin{aligned}0 &= -\frac{2f_1'f_1''f_2'f_2''}{W^4}\Big[\frac{1+f_1^{'2}}{f_1''}+\frac{1+f_2^{'2}}{f_2''}+\frac{1+f_3^{'2}}{f_3''}\Big]+\frac{f_1'f_1''}{W^2}(\frac{1+f_2^{'2}}{f_2''})'\\ &+ \frac{f_2'f_2''}{W^2}(\frac{1+f_1^{'2}}{f_1''})'-\frac{f_1'f_1''f_2'f_2''}{W^4}\Big[\frac{1+f_1^{'2}}{f_1''}+\frac{1+f_2^{'2}}{f_2''}+\frac{1+f_3^{'2}}{f_3''}\Big]\\ &+ kW(\frac{1+f_1^{'2}+f_2^{'2}}{f_1''f_2''})_{x_1x_2},\end{aligned} \tag{3.10}$$

that is

$$\begin{aligned}&\frac{3f_1'f_1''f_2'f_2''}{W^5}\Big[\frac{1+f_1^{'2}}{f_1''}+\frac{1+f_2^{'2}}{f_2''}+\frac{1+f_3^{'2}}{f_3''}\Big] \\ &\quad= \frac{f_2'f_2''}{W^3}(\frac{1+f_1^{'2}}{f_1''})'+\frac{f_1'f_1''}{W^3}(\frac{1+f_2^{'2}}{f_2''})'+k(\frac{1+f_1^{'2}+f_2^{'2}}{f_1''f_2''})_{x_1x_2}.\end{aligned} \tag{3.11}$$

Differentiating the equation (3.11) with respect to x_3, we have

$$\begin{aligned}&\frac{5f_1'f_1''f_2'f_2''f_3'f_3''}{W^2}\Big[\frac{1+f_1^{'2}}{f_1''}+\frac{1+f_2^{'2}}{f_2''}+\frac{1+f_3^{'2}}{f_3''}\Big] \\ &\quad= f_2'f_2''f_3'f_3''(\frac{1+f_1^{'2}}{f_1''})'+f_1'f_1''f_3'f_3''(\frac{1+f_2^{'2}}{f_2''})'+f_1'f_1''f_2'f_2''(\frac{1+f_3^{'2}}{f_3''})'.\end{aligned} \tag{3.12}$$

Divided by $f_1'f_1''f_2'f_2''f_3'f_3''$ on both sides of (3.12), we have

$$\begin{aligned}&\frac{5}{W^2}\Big[\frac{1+f_1^{'2}}{f_1''}+\frac{1+f_2^{'2}}{f_2''}+\frac{1+f_3^{'2}}{f_3''}\Big] \\ &\quad= \frac{1}{f_1'f_1''}(\frac{1+f_1^{'2}}{f_1''})'+\frac{1}{f_2'f_2''}(\frac{1+f_2^{'2}}{f_2''})'+\frac{1}{f_3'f_3''}(\frac{1+f_3^{'2}}{f_3''})'.\end{aligned} \tag{3.13}$$

Next we denote

$$F=\frac{1+f_1'^2}{f_1''},\qquad G=\frac{1+f_1'^2}{f_2''},\qquad M=\frac{1+f_3'^2}{f_3''}. \tag{3.14}$$

Then (3.13) becomes

$$\frac{5}{W^2}\big[F+G+M\big]=\frac{F'}{f_1'f_1''}+\frac{G'}{f_2'f_2''}+\frac{M'}{f_3'f_3''}. \tag{3.15}$$

Differentiating the equation (3.15) with respect to x_1, we have

$$-\frac{10f_1'f_1''}{W^4}\big[F+G+M\big]+\frac{5}{W^2}F'=\Big(\frac{F'}{f_1'f_1''}\Big)'. \tag{3.16}$$

Divided by $5f_1'f_1''$ on both sides of equation (3.16), we have

$$-\frac{2}{W^4}\big[F+G+M\big]+\frac{1}{W^2}\frac{F'}{f_1'f_1''}=\frac{1}{5f_1'f_1''}\Big(\frac{F'}{f_1'f_1''}\Big)'. \tag{3.17}$$

Differentiating the equation (3.17) with respect to x_2, we have

$$\frac{4f_2'f_2''}{W^6}\big[F+G+M\big]-\frac{1}{W^4}G'-\frac{f_2'f_2''}{W^4}\frac{F'}{f_1'f_1''}=0. \tag{3.18}$$

Divided by $f_2'f_2''$ on both sides of equation (3.18), we have

$$\frac{4}{W^2}\big[F+G+M\big]-\frac{G'}{f_2'f_2''}-\frac{F'}{f_1'f_1''}=0. \tag{3.19}$$

Differentiating the equation (3.19) with respect to x_3 and divided by $f_3'f_3''$, we have

$$\frac{M'}{f_3'f_3''}=\frac{2}{W^2}(F+G+M). \tag{3.20}$$

Since equation (3.15) is symmetric with F, G and M, so we have

$$\frac{F'}{f_1'f_1''}=\frac{G'}{f_2'f_2''}=\frac{2}{W^2}(F+G+M). \tag{3.21}$$

Therefore

$$\frac{F'}{f_1'f_1''}+\frac{G'}{f_2'f_2''}+\frac{M'}{2f_3'f_3''}=\frac{6}{W^2}(F+G+M), \tag{3.22}$$

which together with (3.15) gives $F+G+M=0$ and so F, G, M are constants. Similar to case A.1, we also get case (2) in Theorem 5.

Case B. $f_1''f_2''f_3''=0$. In this case, at least one of f_1'', f_2'' and f_3'' vanishes. Without loss of generality, we assume $f_1''=0$ and hence there exists constant c such that $f_1'=c$, then equation (3.3) becomes

$$f_2''f_3''(1+c^2)=kW\big[(1+c^2+f_3'^2)f_2''+(1+c^2+f_2'^2)f_3''\big]. \tag{3.23}$$

Case B.1. $k = 0$. From (3.23), we have $f_2'' f_3'' = 0$. If $f_2'' = f_3'' = 0$, then the hypersurface M is a hyperplane in $\mathbb{R}^4$ and we get case (1) in Theorem 5 with $f_3 = ax_3 + b$. If one of f_2'' and f_3'' is zero, we get case (1) in Theorem 5.

Case B.2. $k \neq 0$. If $f_2'' = 0$, from (3.23) we have $f_3'' = 0$. Then there exist constants a_i, b_i, $c_i (1 \leq i \leq 3)$ such that $f_1 = a_1 x_1 + a_2, f_2 = b_1 x_1 + b_2, f_3 = c_1 x_1 + c_2$, and the hypersurface M is a hyperplane in $\mathbb{R}^4$. We get the case (1) in Theorem 5.

If $f_2'' \neq 0$, from (3.23) we have $f_3'' \neq 0$. Divided by $f_2'' f_3''$ on both sides of equation (3.23), we have

$$(1 + c^2) = kW\big[\frac{1 + c^2 + f_3'^2}{f_3''} + \frac{1 + c^2 + f_2'^2}{f_2''}\big]. \tag{3.24}$$

Differentiating the equation (3.24) with respect to x_2, we have

$$k\frac{f_2' f_2''}{W^2}\big[\frac{1 + c^2 + f_3'^2}{f_3''} + \frac{1 + c^2 + f_2'^2}{f_2''}\big] = -k(\frac{1 + c^2 + f_2'^2}{f_2''})'. \tag{3.25}$$

Since $k \neq 0$, then by (3.24), equation (3.25) becomes

$$\frac{f_2' f_2''}{W^3}\big[1 + c^2\big] = -k(\frac{1 + c^2 + f_2'^2}{f_2''})'. \tag{3.26}$$

Differentiating the equation (3.26) with respect to x_3, we have

$$\frac{1}{W^5} f_2' f_2'' f_3' f_3'' \big[1 + c^2\big] = 0, \tag{3.27}$$

which implies that $f_2'' f_3'' = 0$ and this is a contradiction. This completes the proof of Theorem 5.

Proof of Theorem 6. Let M^n be a Weingarten hypersurface with $GK = kH$ in $\mathbb{R}^{n+1}$. It follows from (2.1) and (2.2) that

$$\frac{f_1'' f_2'' \cdots f_n''}{(1 + \sum_{i=1}^n f_i'^2)^{(n+2)/2}} = k\frac{\sum_{i=1}^n (1 + \sum_{j\neq i}^n f_j'^2) f_i''}{n(1 + \sum_{i=1}^n f_i'^2)^{3/2}},$$

that is

$$\frac{f_1'' f_2'' \cdots f_n''}{(1 + \sum_{i=1}^n f_i'^2)^{(n-1)/2}} = \frac{k}{n}\sum_{i=1}^n (1 + \sum_{j\neq i}^n f_j'^2) f_i''. \tag{3.28}$$

Assume that $f_1'' f_2'' \cdots f_n'' \neq 0$, dividing (3.28) by $f_1'' f_2'' \cdots f_n''$, we have

$$\frac{1}{(1 + \sum_{i=1}^n f_i'^2)^{(n-1)/2}} = \frac{k}{n}\sum_{i=1}^n \frac{1 + \sum_{j\neq i}^n f_j'^2}{\prod_{j\neq i}^n f_j''}. \tag{3.29}$$

Differentiating the equation (3.29) with respect to $x_1, x_2, \cdots, x_n$ successively. We have

$$(-1)^n n \prod_{i=1}^n (n + 2i - 3)(1 + f_1'^2 + \cdots + f_n'^2)^{-\frac{3n-1}{2}} f_1' f_1'' \cdots f_n' f_n'' = 0. \tag{3.30}$$

This implies that $f_1'f_1''\cdots f_n'f_n''=0$, which contradicts to the assumption $f_1''f_2''\cdots f_n''\neq 0$.

Since $f_1''f_2''\cdots f_n''=0$, there exists at least one $f_i''=0$, we assume that $f_1''=0$ and hence $f_1=ax_1+b$, where a and b are constants. It follows immediately that

$$\begin{aligned} X(x_1,\cdots,x_n) &= (x_1,\cdots,x_n,f_1(x_1)+f_2(x_2)+\cdots+f_n(x_n)) \\ &= (x_1,\cdots,x_n,ax_1+b+f_2(x_2)+\cdots+f_n(x_n)) \\ &= x_1(1,0,\cdots,0,a)+(0,x_2,\cdots,x_n,b+f_2(x_2)+\cdots+f_n(x_n)), \end{aligned} \tag{3.31}$$

which implies that M^n is a cylinder. This completes the proof of Theorem 6.

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References

[1] C. P. Aquino, H. F.de Lima, M.A.L. Velasquez, A new characterization of complete linear weingarten hypersurfaces in real space form, *Pacific J. Math.* **261** (2013), 123–136.

[2] X. L. Chao, On complete spacelike submanifolds in semi-Riemannian space forms with parallel normalized mean curvature vector, *Kodai Math. J.* **34** (2011), 42–54.

[3] X. L. Chao, P. J. Wang, Linear Weingarten hypersurfaces in Riemannian space forms, *Bull. Korean Math. Soc.* **51** (2014), 567–577.

[4] Q. M. Cheng, Complete space-like hypersurfaces of a de Sitter space with $r = kH$, *Mem. Fac. Sci. Kyushu Univ.* **44** (1990), 67–77.

[5] F. Dillen, L. Verstraelen and G. Zafindratafa, A generalization of translation surfaces of Scherk, *Differential Geometry in honor of Radu Rosca, K.U.L.* (1991), 107–109.

[6] H. Li, Global rigidity theorems of hypersurfaces, *Ark. Mat.* **35** (1997) 2, 327–351.

[7] H. Li, Y. J. Suh and G. Wei, Linear Weingarten hypersurfaces in a unit sphere, *Bull. Korean Math. Soc.* **46** (2009) 2, 321–329.

[8] B. P. Lima, N. L. Santos, P. A. Sousa, Translation hypersurfaces with constant scalar curvature into the Euclidean space, *Israel J. Math.* **201** (2014), 797–811.

[9] K. Seo, Translation hypersurfaces with constant curvature in space forms, *Osaka J. Math.* **50** (2013), 631–641.

[10] H. F. Sherk, Bemerkungen über die kleinste Fläche innerhalb gegebener Grenzen, *J. Reine Angew. Math.* **13** (1835), 185–208.

[11] S. C. Shu, Linear Weingarten hypersurfaces in a real space form, *Glasg. Math. J.* **52** (2010), 635–648.

[12] S. C. Shu, Complete space-like hypersurfaces in a de Sitter space, *Bull. Austral. Math. Soc.* **73** (2006), 9–16.

[13] L. Verstraelen, J. Walrave and S. Yaprak, The minimal translation surfaces in Euclidean space, *Soochow J. Math.* **20** (1994), 77–82.

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