

Approximate factorization of matrix polynomials with applications to the synthesis problems

by

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Abstract

We consider a problem of approximate factorization of regular and irregular matrix polynomials. A theorem is proved on the minimum value of the functional. A method and numerical algorithm is offered to solving the considered problem using symbolic calculations that provides high accuracy of the obtained solution. Results of the numerical experiments and comparison with known results are given.

Key Words: Factorization, symbolic calculations, irregular polynomial.

2010 Mathematics Subject Classification: Primary 65Y04,
Secondary 65K15

1 Introduction

Recently with application of advanced computer technologies and development of new applied software packages (including Symbolic Toolbox package MATLAB), the accuracy of the solutions of some classes of problems is essentially increased. For example, in [1], [10] using Symbolic Toolbox tools of MATLAB effective and more precise algorithms for the solution of matrix algebraic Riccati (ARE) and Lyapunov equations are given. It is known that solution of ARE plays the basic role in factorization of the polynomials, that is a key procedure in the solution of different synthesis problems. Thus a question arises: whether is it possible to use Symbolic Toolbox package MATLAB for the solution of the optimal synthesis problems for the systems of stabilization and to raise the accuracy of the obtained solution.

When implementing operations on numeric values arise rounding errors, because the accuracy of the calculations is limited by the number of digits used in each operation. Under repeated operations these errors are accumulated. Operations on symbolic variables can be implemented exactly, as in these cases the

calculations are not made on numbers. As a result no rounding errors arise. The system MATLAB performs calculations only in arithmetic with floating point. These operations are limited to the bit, which also in turn leads to errors. However, development and wide application of high technologies generates more classes of problems requiring the implementation of computing without any error. To achieve this aim Symbolic Math Toolbox procedures included in package MATLAB may be used.

In this paper, based on the symbolic computations, high-accuracy computational procedures are proposed for factorization of the matrix polynomials in regular and irregular cases.

2 Description of the problem.

Suppose that the motion of the object is described by the following linear differential equation with constant coefficients

$$Px = Mu + \psi, \quad (2.1)$$

where $x = [x_1, \dots, x_n]'$ is n -dimensional object coordinate; $u = [u_1, \dots, u_m]'$ is m -dimensional control vector; $\psi = [\psi_1, \dots, \psi_n]'$ is n -dimensional vector of external perturbation, the components of which are stationary random processes with zero mathematical expectation and fraction-rational matrix of spectral densities S_ψ ; P and M are $n \times n$ and $n \times m$ dimensional matrices, elements of which $p_{ij}(p)$ and $m_{ij}(p)$ are operator polynomials of $p = \frac{d}{dt}$.

The problem is: To synthesize a regulator

$$W_0 u = \bar{W}(x + \varphi) \quad (2.2)$$

that makes the closed-loop system (2.1), (2.2) asymptotically stable and gives minimum to the functional

$$J = \langle x' R x \rangle + \langle u' C u \rangle. \quad (2.3)$$

Here we suppose that the vector of the coordinate measurement errors $\varphi = [\varphi_1(t), \varphi_2(t), \dots, \varphi_n(t)]'$ is a vector of stationary random processes with zero mathematical expectation and spectral density S_φ .

Applying Laplace transformation to the equations (2.1)-(2.2) we get

$$P(s)x(s) = M(s)u(s) + \psi(s), \quad (2.4)$$

$$u(s) = W(s)[x(s) + \varphi(s)], \quad (2.5)$$

where $W(s)$ is defined as

$$W(s) = W_0^{-1}(s)\bar{W}(s). \quad (2.6)$$

The functions $x(s)$ and $u(s)$ may be expressed by the relations

$$\left. \begin{aligned} x(s) &= F_x^\psi \psi(s) + F_x^\varphi \varphi(s) \\ u(s) &= F_u^\psi \psi(s) + F_u^\varphi \varphi(s) \end{aligned} \right\}, \quad (2.7)$$

where

$$\left. \begin{aligned} F_x^\psi &= (P - MW)^{-1} \\ F_u^\psi &= W(P - MW)^{-1} \\ F_x^\varphi &= (P - MW)^{-1}P - E \\ F_u^\varphi &= W(P - MW)^{-1}P \end{aligned} \right\}. \quad (2.8)$$

Using Fourier transformation and taking $s = i\omega$ [3] the functional (2.3) may be written as follows

$$\begin{aligned} J &= \frac{1}{i} \int_{-i\infty}^{i\infty} Sp \left[(F_{x*}^\psi R F_x^\psi + F_{u*}^\psi C F_u^\psi) S_\psi + \right. \\ &\quad \left. + (F_{x*}^\varphi R F_x^\varphi + F_{u*}^\varphi C F_u^\varphi) S_\varphi \right] ds, \end{aligned} \quad (2.9)$$

where “*” stands for the operation of transposing and replacement s by $-s$. Thus, the problem is reduced to the finding of the matrix W that, provides minimum to the functional (2.9) and makes stable the closed-loop “object-regulator” system. Using (2.8) we obtain

$$\left. \begin{aligned} F_x^\varphi &= F_x^\psi P - E \\ F_u^\varphi &= F_x^\psi P \end{aligned} \right\}. \quad (2.10)$$

If to consider (2.7) in (2.4) we obtain that the functions F_x^ψ and F_u^ψ satisfy the condition

$$P F_x^\psi - M F_u^\psi = E_n. \quad (2.11)$$

Introduce $m \times n$ matrix Φ

$$A F_x^\psi + B F_u^\varphi = \Phi, \quad (2.12)$$

where A and B are polynomial matrices of dimensions $m \times n$ and $m \times m$, respectively [9].

Then the matrix W of the transition functions of the regulator may be found as

$$W = (B + \Phi M)^{-1} (\Phi P - A). \quad (2.13)$$

If to denote

$$\left. \begin{aligned} \Delta_p(s) &= \det P, \\ N &= \Delta_p(s) P^{-1} M, \\ Q &= \Delta_p(s) B + A N \end{aligned} \right\} \quad (2.14)$$

and factorize the functions

$$\left. \begin{aligned} \bar{D} &= S_\psi + P S_\varphi P_* \\ \bar{G} &= N_* R N + \Delta_p^*(s) C \Delta_p(s) \end{aligned} \right\} \quad (2.15)$$

as

$$\left. \begin{aligned} \bar{D} &= DD_* \\ \bar{G} &= G_*G \end{aligned} \right\}, \quad (2.16)$$

then the functional (2.9) takes the form [3]

$$\begin{aligned} J = & \frac{1}{i} \int_{-i\infty}^{i\infty} S_p \left\{ \Phi_* Q_*^{-1} G_* Q_*^{-1} \Phi DD_* + \Phi_* [Q_*^{-1} (NR - G_* G Q_*^{-1} A) P^{-1} DD_* - \right. \\ & - Q_*^{-1} N_* R S_\varphi P_*] + [DD_* P_*^{-1} (RN - A_* Q_*^{-1} G_* G) Q_*^{-1} - PS_\varphi RN Q_*^{-1}] \Phi + \\ & + P_*^{-1} (R - A_* Q_*^{-1} N_* R - RN Q_*^{-1} A) P^{-1} S_\psi + \\ & \left. + P_*^{-1} A_* Q_*^{-1} G_* G Q_*^{-1} A P^{-1} DD_* \right\} ds. \end{aligned} \quad (2.17)$$

Denote

$$Z = \begin{bmatrix} P & -M \\ A & B \end{bmatrix}. \quad (2.18)$$

Then the equations (2.11), (2.12) may be written as follows

$$Z \cdot \begin{bmatrix} F_x^\psi \\ F_u^\psi \end{bmatrix} = \begin{bmatrix} E_n \\ \Phi \end{bmatrix}. \quad (2.19)$$

From the last we obtain the solution

$$\begin{bmatrix} F_x^\psi \\ F_u^\psi \end{bmatrix} = Z^{-1} \begin{bmatrix} E_n \\ \Phi \end{bmatrix}. \quad (2.20)$$

In order to the closed-loop system be stable, the matrices Z and Z^{-1} should be analytic in the right half plane.

Now let's find the matrix Φ that minimizes the functional (2.17). For this purpose we factorize the functions

$$\left. \begin{aligned} \bar{D} &= S_\psi + PS_\varphi P_* \\ \bar{H} &= Q_*^{-1} G_* G Q_*^{-1} \end{aligned} \right\} \quad (2.21)$$

as follows

$$\left. \begin{aligned} \bar{D} &= DD_*, \\ \bar{H} &= H_* H. \end{aligned} \right\} \quad (2.22)$$

Then we get

$$\bar{K} = H_*^{-1} (Q_*^{-1} N_* R - H_* H A) P^{-1} D \quad (2.23)$$

and

$$\bar{L} = -H_*^{-1} Q_*^{-1} N_* R S_\varphi P_* D^{-1}. \quad (2.24)$$

If to separate the functions \bar{K} and \bar{L} we have

$$\bar{K} = K_0 + K_+ + K_- , \quad (2.25)$$

$$\bar{L} = L_0 + L_+ + L_- \tag{2.26}$$

Here K_0 and L_0 are integer parts of the fractional expressions \bar{K} and \bar{L} , correspondingly. K_+ and L_+ are proper fractions with poles only in the left half plane, K_- and L_- -with poles in the right half plane. Then the matrix Φ making zero the first variation of the functional (2.17) and having poles only in the left half plane will be defined by the formula

$$\Phi = -H^{-1}(K_0 + K_+ + L_0 + L_+)D^{-1}. \tag{2.27}$$

If to consider this in (2.17), then the validity of the following theorem is proved.

Theorem 1. *For the minimum of the functional (2.3) subject to (2.1) and (2.2) the following formula holds true*

$$\begin{aligned} J_{\min} = & \frac{1}{i} \int_{-i\infty}^{i\infty} \{Sp[(K_- + L_-)(K_- + L_-)_*] + \\ & + Sp [P^{-1}S_\psi P_*^{-1}(R - RNG^{-1}G_*^{-1}N_*R) + \\ & + S_\varphi RNG^{-1}G_*^{-1}N_*R(E_n - S_\varphi P_* D_*^{-1}D^{-1}P)]\} ds. \end{aligned} \tag{2.28}$$

Thus as we see from these considerations, factorization is the key procedure in the solution of the above stated problem and accuracy of solution of the last one mainly depends on the accuracy of the factorization procedure. Note that these problems have been considered by various authors in different formulations [3, 6, 7, 12]. But development of the high technologies and their applications in different fields makes necessary to obtain more precise solutions. Considering this we offer below high accuracy calculation algorithms for the factorization of the regular and irregular matrix polynomials.

3 Factorization of the matrix polynomials with respect to the imaginary axis.

Let's consider the regular matrix polynomial

$$A(s) = (-1)^n E s^{2n} + (-1)^{n-1} A_1 s^{2n-2} + \dots + A_{2n}, \tag{3.1}$$

where $A_i > 0$, $i = \overline{1, 2n}$. Here $A_i = A'_i$ for even, and $A_i = -A'_i$ for odd indexes, A_i - constant $m \times m$ dimensional matrices, the sign " ' " means the operation of transpose, E is unit matrix.

The problem is: Find a matrix polynomial $H(s)$, satisfying the relation

$$A(s) = H'(-s)H(s) \tag{3.2}$$

under the condition that the zeros of $H(s)$ lie on the left half-plane and $H'(-s)$ - in the right half-plane and $H^{-1}(s)$ has no poles in the right half plane.

The solution of the problem (3.2) we present as

$$H(s) = Es^n + (L' + G'X)N, \tag{3.3}$$

where X is a positively defined solution of ARE

$$XF + F'X - XGG'X + R = 0, \tag{3.4}$$

with

$$G = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ E_m \end{bmatrix}, N = \begin{bmatrix} E_m \\ E_m s \\ \dots \\ \dots \\ E_m s^{n-1} \end{bmatrix},$$

$$F = \begin{bmatrix} 0 & E_m & 0 & \dots & 0 \\ 0 & 0 & E_m & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot \\ 0 & 0 & 0 & \dots & E_m \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, L = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ 0 \\ \frac{(-1)^{n-1}}{2} A_1 \end{bmatrix}, \tag{3.5}$$

$$R = \begin{bmatrix} A_{2n} & \frac{1}{2} A_{2n-1} & 0 & \dots & 0 \\ \frac{1}{2} A'_{2n-1} & -A_{2n-2} & -\frac{1}{2} A_{2n-3} & \dots & 0 \\ 0 & & & & \vdots \\ \vdots & & & & \frac{(-1)^{n-2}}{2} A_3 \\ 0 & \dots & \frac{(-1)^{n-2}}{2} A'_3 & (-1)^{n-1} A_2 & \end{bmatrix}.$$

To provide absence of the poles of $H^{-1}(s)$ in the right half-plane we choose the solution of (3.4) by which $F - G(L' + G'X)$ is a Hurwitz matrix, i.e. its eigenvalues lie in the left half plane.

There exist various methods to solving (3.4). We can note Shur’s method, method of infinite numbers, matrix signum-function method etc. [2, 4, 5]. Considering that the last method may be realized on Symbolic calculations, we’ll use that one.

Algorithm for the computation of the matrix sign-function is as follows [8].

Algorithm 1.

Input: Matrix B of dimension $n \times n$;

Output: Sign-function of the matrix B ;

Step 1. Take $B_0 = B$;

Step 2. Calculate

$$\alpha = |\det B_i|^{\frac{1}{n}},$$

$$B_{i+1} = \frac{1}{2} (B_i + \alpha^2 B_i^{-1});$$

for $i = 0, 1, \dots$

Step 3. Check up the criteria

$$\|B_{i+1} - B_i\| < \varepsilon,$$

where ε is needed accuracy, $\|B\|$ is matrix norm defined by its maximal element.

Realization of this algorithm may be found in Symbolic Toolbox package MATLAB (procedure `signm.m`).

Now we can use this algorithm for solving ARE (3.4).

Algorithm 2.

Input: Matrices: F of dimension $n \times n$, G of dimension $n \times m$, R of dimension $n \times n$, ε - needed accuracy;

Output: Matrix $X = X' > 0$;

Step 1. Calculate $S = GG'$;

Step 2. Form the auxiliary matrix

$$A = \begin{bmatrix} F & S \\ -R & -F' \end{bmatrix}.$$

Step 3. Calculate $\text{sign}A$ by Algorithm 1;

Step 4. Determine required X by the relation

$$(\text{sign}A + E) \cdot X = 0.$$

The software for the realization of this algorithm on Symbolic Toolbox is developed.

Therefore the algorithm of factorization of polynomial (3.1) is as follows.

Algorithm 3.

Input: A_0, A_1, \dots, A_{2n} - coefficients of the given polynomial;

Output: H_0, H_1, \dots, H_n - coefficients of the seeking polynomial;

Step 1. Form the matrices F, G, R according to (3.5);

Step 2. Solve ARE (3.4) by Algorithm 2;

Step 3. Calculate the coefficients H_0, H_1, \dots, H_n of the polynomial $H(s)$ according to (3.3);

Step 4. Define $\bar{A}(s) = H(s) \cdot H^*(s)$;

Step 5. Check up the condition $\|A(s) - \bar{A}(s)\| < \varepsilon$. If it is satisfied, then the calculation stops.

Otherwise take the coefficients of the polynomial $\bar{A}(s)$ as initial data and go to Step 1.

4 Factorization of the irregular matrix polynomial.

Let the matrix polynomial be given

$$B(s) = (-1)^n B_0 s^{2n} + B_1 s^{2n-1} + \dots + B_{2n}, \quad (4.1)$$

where B_0 has no inverse, i.e. $B(s)$ is an irregular matrix polynomial. Here $B_i = B'_i$ for even and $B_i = -B'_i$ for odd indexes.

Our aim is to find such matrix-polynomial $D(s)$, that

$$B(s) = D'(-s)D(s).$$

Usually, the computational procedure for solving this problem consists in eliminating the roots of $\det B(s)$, by successive multiplication the polynomial $B(s)$ on the left and right by the corresponding matrices until one obtains a constant matrix whose factorization is trivial.

Thus the factorization in this case includes the following steps.

1. Choose special matrix polynomial $T(s)$ that transfers $B(s)$ into regular matrix, i.e. $T_*(s)B(s)T(s) = A(s)$, where $A(s)$ is regular.
2. Use Algorithm 3 to factorize $A(s)$

$$A(s) = H_*(s) \cdot H(s). \quad (4.2)$$

3. Find the sought matrix $D(s)$ from $D(s) = H(s)T^{-1}(s)$.

Let us illustrate the scheme of realization of the first step.

For the realization of this step the standard procedure *svd* (singular vector decomposition) from the MATLAB Symbolic Toolbox is used $[U, D, V] = svd(A)$ where $A = U D V'$.

Multiplying $B(s)$ by V , we get the polynomial

$$V' B(s) V = [B_{ij}(s)], \quad ij = 1, \dots, m.$$

Then the matrix $T_1(s)$ is defined in the form

$$T_1(s) = \text{diag}\{1, \dots, 1, (s + \alpha_1)^{\delta_1}, 1, \dots, 1\},$$

where δ_m is the maximal order of the out off diagonal polynomial. After finite time of cycles we arrive to the regular polynomial

$$\bar{B}(s) = T_{0*}(s)B(s)T_0(s) = (-1)^n \bar{B}_{01}s^{2n} + \dots + \bar{B}_{02n}. \quad (4.3)$$

Here the matrix \bar{B}_{01} is positively defined, i.e. it may be presented in the form $\bar{B}_{01} = Q'Q$, where Q is upper triangle matrix. To get this representation the procedure *chol(A)* (Cholesky decomposition) from the Symbolic Toolbox of Matlab is used. Multiplying $\bar{B}(s)$ from the left by $(Q')^{-1}$ and from the right by Q^{-1} one obtains the matrix polynomial with the unit matrix as a coefficient at the high order

$$\bar{B}(s) = (-1)^n E s^{2n} + A_1 s^{2n-1} + \dots + A_{2n}, \quad (4.4)$$

where $A_j = (Q')^{-1} \bar{B}_{0j} Q^{-1}$, $j = 1, 2, \dots, 2n$.

In the second step of the algorithm the matrix $H(s)$, factorizing the polynomial (4.4) is defined. In this step matrix algebraic Riccati equation is solved using the matrix signum function method described above.

In the third step the seeking matrix $D(s)$ is defined as

$$D(s) = H(s)QT_j^{-1}(s)V'_\nu \cdots T_1^{-1}(s)V'_1.$$

Here the operation of multiplication the matrix from the right by the $T_j^{-1}(s)$ in fact consists of the division of the polynomials of the corresponding columns of this matrix by the polynomial $(s + \alpha_j)^{\delta_j}$ standing for the diagonal of the matrix $T_j(s)$. For the sake of simplicity of calculations let us take $\alpha_j = 1$.

The package MATLAB includes the standard procedures of division and multiplication of polynomials. The arguments of these operations are numerical variables. Those procedures do not provide the results with the necessary accuracy, and do not support the Symbolic Toolbox of MATLAB. Considering this we developed software for multiplication and division of polynomials that supports Symbolic Toolbox of MATLAB.

Let's illustrate the realization of this algorithm for the irregular matrix polynomial on the example below.

Example 1. It needs to factorize the following matrix polynomial

$$B(s) = \begin{bmatrix} 3 - s^2 & s^4 \\ s^4 & 2 + s^8 \end{bmatrix}.$$

For this case in (4.1) we have

$$B_1 = B_2 = B_3 = B_5 = B_7 = 0,$$

$$B_0 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, B_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, B_6 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, B_8 = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}.$$

The matrix $U = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ transforms B_0 into a diagonal form. Multiplying the polynomial $B(s)$ from the left and right by U we obtain

$$U'B(s)U = \begin{bmatrix} 2 + s^8 & -s^4 \\ -s^4 & 3 - s^2 \end{bmatrix}.$$

The matrix $T(s)$ is defined in the form $T(s) = \text{diag}\{1, (s + 1)^3\}$. Calculating the matrix polynomial

$$T_*(s)U'B(s)UT(s) = \begin{bmatrix} 2 + s^8 & -s^4(1 + s)^3 \\ -s^4(1 - s)^3 & (3 - s^2)(1 - s^2)^3 \end{bmatrix},$$

we came to the following regular matrix polynomial

$$A(s) = A_1s^8 + A_2s^7 + A_3s^6 + A_4s^5 + A_5s^4 + A_6s^3 + A_7s^2 + A_8s + A_9,$$

where

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 3 \\ 3 & -6 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}, A_5 = \begin{bmatrix} 0 & 1 \\ 1 & 12 \end{bmatrix}, A_6 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$A_7 = \begin{bmatrix} 0 & 0 \\ 0 & -10 \end{bmatrix}, A_8 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, A_9 = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}.$$

Factorization $A(s) = H'(-s)H(s)$ gives

$$H_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 2.6702467521558406665425481245717 & -0.21388401189504250760922278296807 \\ 0.78611598810495749239077721703342 & 4.7187942370905486937889886190026 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 3.8740980320765246205111837077708 & -0.64165203568512752282766834890364 \\ 0.78004854193874481711880311069832 & 8.1563827112716460813669658570098 \end{bmatrix},$$

$$H_3 = \begin{bmatrix} 3.2990762000942031797723236099326 & -0.64165203568512752282766834890411 \\ 0.51213530895998361311306354355974 & 6.1563827112716460813669658570098 \end{bmatrix},$$

$$H_4 = \begin{bmatrix} 1.4033896179027123174615514817531 & -0.21388401189504250760922278296781 \\ 0.17463556442740729223783760919692 & 1.7187942370905486937889886190027 \end{bmatrix}.$$

The accuracy of these calculations is $3.224839186938058e - 029$.

Thus the seeking matrix has a form

$$D_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, D_2 = \begin{bmatrix} 0 & 2.6702467521558406665425481245717 \\ 0 & 0.78611598810495749239077721703342 \end{bmatrix},$$

$$D_3 = \begin{bmatrix} 0 & 3.8740980320765246205111837077708 \\ 0 & 0.78004854193874481711880311069832 \end{bmatrix},$$

$$D_4 = \begin{bmatrix} 0 & 3.2990762000942031797723236099326 \\ 1 & 0.51213530895998361311306354355974 \end{bmatrix},$$

$$D_5 = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix},$$

where

$$d_{11} = 0.21388401189504250760922278296807,$$

$$d_{12} = 1.4033896179027123174615514817531,$$

$$d_{21} = -1.7187942370905486937889886190026,$$

$$d_{22} = 0.17463556442740729223783760919692.$$

The coefficients of the irregular matrix polynomial obtained in result of multiplication $\bar{B}(s) = D_*(s)D(s)$ differ from the initial ones in the 29th digit. Thus the considered problem is solved with accuracy of $1.577822734559815e-029$.

Now let's consider the comparable investigation of one dimensional synthesis problem on the following example.

Example 2. Consider the example given in [11, page 81], where one dimensional synthesis problem is studied. In our denotations the data of that example indeed are:

$$P = s^2 - 1, \quad M = 1, \quad R = \frac{r_1}{r_2} = \frac{9}{9 - s^2}, \quad C = \frac{c_1}{c_2} = \frac{10^{-4}}{1},$$

$$S_\psi = \frac{S_{1\psi}}{S_{2\psi}} = \frac{4}{4-s^2}, \quad S_\varphi = \frac{S_{1\varphi}}{S_{2\varphi}} = \frac{10^{-8}}{1}.$$

For this problem in [11] the following values are obtained

$$W_0^T = [1 \quad 68.2926 \quad 2324.43 \quad 10863.32 \quad 13432.718],$$

$$W_1^T = [35032.913 \quad 418783.21 \quad 22839.57 \quad 6013724.93]$$

with coefficients

$$q_1 = [10^{-4} \quad 5.43610^{-3} \quad 0.14746 \quad 1.9999],$$

$$g_1 = [10^{-2} \quad 0.1393 \quad 0.9153 \quad 3.0002].$$

Calculating the optimality condition we get $nevv = 7.910523150899287e + 001$. As one can see from this the obtained regulators are far from the optimality ones.

Now we apply above given procedures with symbolic calculations and get the following values for the coefficients

$$q_1 = [1.000000000000000e - 004 \quad 5.436203300704970e - 003$$

$$1.474615316329790e - 001 \quad 2.000000010000019e + 000],$$

$$g = [1.000000000000000e - 002 \quad 1.393054311171015e - 001$$

$$9.153001569360758e - 001 \quad 3.000149996250183e + 000],$$

$$W_0 = [9.999999999999998e - 001 \quad 6.829257611875983e + 001$$

$$2.324437976468313e + 003 \quad 1.086533336817375e + 004$$

$$1.343630050975466e + 004],$$

$$W_1 = [3.502092427916212e + 004 \quad 4.187799894518985e + 005$$

$$2.283872369969072e + 006 \quad 6.013736323011676e + 006].$$

The optimality condition $nevv = 0.1398e - 035$ is satisfied.

Comparing the coefficients of the corresponding regulator we get

$$\|W_0^T - W_0\| = 4.1095, \quad \|W_1^T - W_1\| = 12.7862.$$

5 Conclusion.

In the work the algorithms are proposed for the factorization of the regular and irregular matrix polynomials. Use of symbolic calculations provides high accuracy of the obtained solutions. The examples are given that demonstrate the efficiency of the proposed methods and comparison with known results is provided.

Acknowledgement. The work is supported by the Baku State University "50+50" Grant, No.1-57.

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Received: 19.01.2015

Revised: 06.04.2015

Accepted: 28.05.2015

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