Bull. Math. Soc. Sci. Math. Roumanie
Tome 57(105) No. 1, 2014, 121–130

Asymptotic stability of solutions of a class of neutral differential equations with multiple deviating arguments

by
Cemil Tunç

Abstract

In this paper, using the Lyapunov-Krasovskii functional approach, some novel stability criteria are presented for all solutions of a class of nonlinear neutral differential equations to tend zero when $t \to \infty$.

Key Words: Neutral differential equation; asymptotic stability; deviating argument; Lyapunov-Krasovskii functional.
2010 Mathematics Subject Classification: Primary 34K20,
Secondary 34K40.

1 Introduction

The stability analysis of neutral systems that involve time-delay in both state and state derivative simultaneously has been widely investigated by many researchers and is still being investigated due to the their applications in many scientific and engineering fields, such as, aircraft, chemical and process control systems, biological systems and economics and so on (see, for example, Agarwal and Grace [1], Charitonov and Melechor-Aguilar [2], El-Morshedy and Gopalsamy [3], Fridman, ([4], [5]), Fridman and Shaked [6], Gopalsamy [7], Gopalsamy and Leung [8], Gu [9], Gyori and Ladas [10], Hale [11], Hale and Verduyn Lunel [12], Kolamnovskii and Myshkis [13], Kolmanovskii and Nosov [14], Liao and Wong [15], Logemann and Townley [16], Nam and Phat [17], P. Park [18], J. H. Park [19], Shaojiang Deng et al. [20], Sun and Wang [21], Tunç and Sirma [22] and the references cited in these sources).

Besides, in 2000, El-Morshedy and Gopalsamy [3] considered the neutral differential equation of the form

$$\frac{d}{dt}[x(t) + px(t-\tau)] = -ax(t) + b\tanh x(t-\tau), t \geq 0, \tag{1.1}$$

where $a,\ b$ are positive real numbers, $\tau \geq 0$ and p is a real number such that $|p| < 1$. For each solution $x(t)$ of Eq. (1), we assume the initial condition

$$x(t) = \phi(t), t \in [-\tau, 0], \phi \in C([-\tau, 0],\ \Re).$$

In this work, first, instead of Eq. (1.1), we consider a nonlinear neutral differential equation with constant deviating argument τ of the form

$$\frac{d}{dt}[x(t) + px(t-\tau)] = -h(x(t))x(t) + q(t)\tanh x(t-\tau), t \geq 0, \tag{1.2}$$

where τ and p are real numbers such that $\tau \geq 0$ and $|p| < 1$, $h : \Re \to \Re^+$, $\Re^+ = (0, \infty)$, is a continuous function.

Here some new sufficient conditions for the asymptotic stability of solutions of Eq. (1.2) are given by the Lyapunov- Krasovskii functional approach. Our purpose is to study the problem of asymptotic stability of solutions of Eq. (1.2). The first part of this paper is motivated by the work of El-Morshedy and Gopalsamy [3] and the papers mentioned above. We obtain some sufficient conditions for the asymptotic stability of solutions of Eq. (1.2). It is followed that the equation discussed by El-Morshedy and Gopalsamy [3], Eq. (1.1), is a special case of our equation, Eq. (1.2). That is, our equation, Eq. (1.2), includes Eq. (1.1) discussed by El-Morshedy and Gopalsamy [3]. By this work, we first generalize a result in [3].

At the same time, in 2000, Agarwal and Grace [1] considered the neutral differential equation of the form

$$\frac{d}{dt}[x(t) + c(t)x(t-\tau)] + p(t)x(t) = q(t)\tanh x(t-\sigma), t \geq 0, \tag{1.3}$$

where τ and σ are positive real numbers, $\sigma \geq \tau$, c, p, $q : [t_0, \infty) \to [0, \infty)$ are continuous, and $c(t)$ is differentiable with local bounded derivative.

The asymptotic stability of Eq. (1.3) when $c(t) = 0$ has also been discussed by many authors. Here, we referee the readers to the monographs of Gopalsamy [7], the books of Gyori and Ladas [10], Hale [11] and the references therein.

In this work, second, instead of Eq. (1.3), we consider a nonlinear neutral differential equation with multiple deviating arguments of the form

$$\frac{d}{dt}[x(t) + \sum_{i=1}^{2} c_i(t)x(t-\tau_i)] + p(t)x(t) = q(t)\tanh x(t-\sigma), t \geq 0, \tag{1.4}$$

where τ_i, $(i = 1,\ 2)$, and σ are positive real numbers, $\sigma \geq \tau_i$, c_i, p, $q : [t_0, \infty) \to [0, \infty)$ are continuous functions, and $c_i(t)$ are differentiable with local bounded derivative.

At the end, we study the asymptotic stability of solutions of Eq. (1.4). The second part of this paper is motivated by the work of Agarwal and Grace [1] and the papers mentioned above. We obtain some sufficient conditions for the asymptotic stability of solutions of Eq. (1.4). It is seen that the equation discussed by Agarwal and Grace [1], Eq. (1.3), is a special case of our equation, Eq. (1.4). That is, our equation, Eq. (1.4), includes Eq. (1.4) discussed by Agarwal and Grace [1]. By this work, we improve a result of Agarwal and Grace [1, Theorem 3] for an equation with multiple deviating arguments, Eq. (1.4).

2 Main results

Theorem 1. *In addition to the basic assumption imposed on the functions h and q that appearing in Eq. (1.2), we assume that there exist constants a_0, a, b and p such that the* *conditions*

$$0 < a_0 \le h(x(t)) \le a, 0 < q(t) \le b$$

and

$$a_0^2 - a^2p^2 > b^2$$

hold. Then, every solution $x(t)$ of Eq. (1.2) satisfies $x(t) \to 0$ as $t \to \infty$.

Proof: Define the Lyapunov- Krasovskii functional

$$V_1(x(t)) = [x(t) + px(t-\tau)]^2 + \int\limits_{t-\tau}^{t} h(x(s))x^2(s)ds. \tag{2.1}$$

The main tool for the proof of Theorem 1 is the Lyapunov- Krasovskii functional given by (2.1).

One can calculate the time derivative of V_1 along solutions of (1.2) as follows

$$\begin{aligned}
\frac{dV_1}{dt} &= 2[x(t)+px(t-\tau)][-h(x(t))x(t)+q(t)\tanh x(t-\tau)] \\
&\quad +h(x(t))x^2(t) - h(x(t-\tau))x^2(t-\tau) \\
&= -h(x(t))x^2(t) + 2q(t)x(t)\tanh x(t-\tau) \\
&\quad -2ph(x(t))x(t)x(t-\tau) + 2pq(t)x(t-\tau)\tanh x(t-\tau) \\
&\quad -h(x(t-\tau))x^2(t-\tau) \\
&= -h(x(t))\left[x^2(t) + \frac{2q(t)\tanh x(t-\tau) - 2ph(x(t))x(t-\tau)}{-h(x(t))}x(t)\right] \\
&\quad +2pq(t)x(t-\tau)\tanh x(t-\tau) - h(x(t-\tau))x^2(t-\tau) \\
&= -h(x(t))\left[x(t) + \frac{q(t)\tanh x(t-\tau) - ph(x(t))x(t-\tau)}{-h(x(t))}\right]^2 \\
&\quad +h(x(t))\left[\frac{q(t)\tanh x(t-\tau) - ph(x(t))x(t-\tau)}{-h(x(t))}\right]^2 \\
&\quad +2pq(t)x(t-\tau)\tanh x(t-\tau) - h(x(t-\tau))x^2(t-\tau) \\
&\le h(x(t))\left[\frac{q(t)\tanh x(t-\tau) - ph(x(t))x(t-\tau)}{-h(x(t))}\right]^2 \\
&\quad +2pq(t)x(t-\tau)\tanh x(t-\tau) - h(x(t-\tau))x^2(t-\tau) \\
&= \frac{q^2(t)\tanh^2 x(t-\tau) + p^2h^2(x(t))x^2(t-\tau) - 2pq(t)h(x(t))x(t-\tau)\tanh x(t-\tau)}{h(x(t))} \\
&\quad +2pq(t)x(t-\tau)\tanh x(t-\tau) - h(x(t-\tau))x^2(t-\tau).
\end{aligned}$$

Using the assumptions of Theorem 1 and the estimate $\tanh^2 x(t) \le x^2(t)$, we get

$$\begin{aligned}
\frac{dV_1}{dt} &\le \frac{q^2(t)x^2(t-\tau) + p^2h^2(x(t))x^2(t-\tau) - 2pq(t)h(x(t))x(t-\tau)\tanh x(t-\tau)}{h(x(t))} \\
&\quad +2pq(t)x(t-\tau)\tanh x(t-\tau) - h(x(t-\tau))x^2(t-\tau) \\
&= \left[\frac{q^2(t) + p^2h^2(x(t))}{h(x(t))} - h(x(t-\tau))\right] x^2(t-\tau) \\
&\le \left[\frac{b^2 + a^2p^2}{a_0} - a_0\right] x^2(t-\tau) \\
&\le \left[\frac{b^2 + a^2p^2 - a_0^2}{a_0}\right] x^2(t-\tau) \le 0, \quad t > 0. \qquad (2.2)
\end{aligned}$$

Hence, integrating both sides of (2.2) from 0 to t, it follows that

$$V_1(x(t)) + \frac{a_0^2 - b^2 - a^2p^2}{a_0} \int_{-\tau}^{t-\tau} x^2(s)ds \le V(x(0)), \quad t > 0. \qquad (2.3)$$

Since $V_1(x(t)) \ge 0$ and $a_0^2 - a^2p^2 > b^2$, then the above estimate leads

$$\int_{-\tau}^{\infty} x^2(s)ds < \infty. \qquad (2.4)$$

Using Minkowski's integral inequality, (2.4) implies

$$\begin{aligned}
\int_0^t [x(s) + px(s-\tau)]^2 ds &\le \left[\left(\int_0^t x^2(s)ds\right)^{\frac{1}{2}} + |p| \left(\int_0^t x^2(s-\tau)ds\right)^{\frac{1}{2}}\right]^2 \\
&\le \left[\left(\int_0^\infty x^2(s)ds\right)^{\frac{1}{2}} + |p| \left(\int_0^\infty x^2(s-\tau)ds\right)^{\frac{1}{2}}\right]^2.
\end{aligned}$$

Hence, we get

$$\int_0^\infty [x(s) + px(s-\tau)]^2 ds < \infty.$$

On the other hand, we conclude from (2.3) that $V_1(x(t))$ is bounded. Therefore, there exists a positive real number M such that

$$[x(t) + px(t-\tau)]^2 < M^2 \quad \text{for all } t \ge 0.$$

Hence,

$$|x(t) + px(t-\tau)| < M \quad \text{forall } t \ge 0,$$

which implies

$$|x(t)| < |p| \;\; |x(t-\tau)| + M \quad \text{forall } t \geq 0. \tag{2.5}$$

We claim that $|x(t)|$ is bounded. Suppose that $|x(t)|$ is not bounded. Then there exists a subsequence $\{t_k\}$, $t_k \to \infty$ as $k \to \infty$ such that

$$|x(t_k)| = \sup\{|x(t)| : t \leq t_k\}, k = 1, 2, \dots .$$

Clearly, it follows that

$$\lim_{k\to\infty} |x(t_k)| = \infty \text{ and } |x(t_k)| \geq |x(t_k - \tau)| \quad \text{for } k = 1, 2, \dots .$$

Hence, (2.5) leads

$$|x(t_k)| < |p| \;\; |x(t_k - \tau)| + M < |p| \;\; |x(t_k)| + M, k = 1, 2, \dots .$$

Then, we have

$$|x(t_k)| < \frac{M}{1 - |p|}.$$

Thus

$$\infty < \frac{M}{1 - |p|}, \quad \text{when } k \to \infty.$$

This case is a contradiction, and implies that $|x(t)|$ is bounded. On the other hand, we observe

$$\frac{d}{dt}[x(t) + px(t-\tau)]^2 = 2[x(t) + px(t-\tau)] \; [-h(x(t))x(t) + q(t)\tanh x(t-\tau)].$$

Then $\frac{d}{dt}[x(t)+px(t-\tau)]^2$ is bounded, which means that $[x(t)+px(t-\tau)]^2$ is uniform continuous. This completes all the requirements of Barbalat's lemma [7, Lemma 1.2.2]. Hence

$$\lim_{t\to\infty} (x(t) + px(t-\tau))^2 = 0$$

or, equivalently,

$$\lim_{t\to\infty} x(t) + px(t-\tau) = 0.$$

Since the assumptions of Theorem 1 implies that $|p| < 1$, by Lemma 1.5.1 of [10] and the foregoing limit, we conclude

$$\lim_{t\to\infty} x(t) = 0$$

which is our desired conclusion. □

Finally, some new sufficient conditions for the asymptotic stability of solutions of Eq. (1.4) are established by the Lyapunov- Krasovskii functional approach.

Theorem 2. *In addition to the basic assumption imposed on thefunctions c_i, p and q that appearing in Eq. (1.4), we assume that there exist positive constants α, β_i and non-negative real numbers c_i, p_1, p_2, q_1 and q_2 such that the following conditions hold:*

$$p_1 \le p(t) \le p_2, q_1 \le q(t) \le q_2, c_i \le c_i(t) \le c_{i+1} < 1, \alpha, \quad \beta_i > q_2 c_{i+1}$$

and

$$2p_1 \ge 2\alpha + \beta_1 + \beta_2 + \frac{p_2^2 c_2}{\alpha - q_2 c_2} + \frac{p_2^2 c_2}{\alpha - q_2 c_3} + \frac{q_2^2}{\sum\limits_{i=1}^{2} \beta_i - q_2 \sum\limits_{i=1}^{2} c_{i+1}}.$$

Then, every solution $x(t)$ of Eq. (1.4) satisfies $x(t) \to 0$ as $t \to \infty$.

Proof: Define the Lyapunov- Krasovskii functional

$$V_2(t) = [x(t) + \sum_{i=1}^{2} c_i(t)x(t-\tau_i)]^2 + \alpha \sum_{i=1}^{2} \int\limits_{t-\tau_i}^{t} x^2(s)ds + \sum_{i=1}^{2} \beta_i \int\limits_{t-\sigma}^{t} \tanh^2 x(s)ds, \tag{2.6}$$

where $\alpha > 0$ and $\beta_i > 0$ are some real numbers.

The main tool for the proof of Theorem 2 is the Lyapunov- Krasovskii functional mentioned above, in (2.6).

One can calculate the time derivative of V_2 along solutions of Eq. (1.4) as follows

$$\begin{aligned}
\frac{dV_2(t)}{dt} &= 2[x(t) + \sum_{i=1}^{2} c_i(t)x(t-\tau_i)][q(t)\tanh x(t-\sigma) - p(t)x(t)] \\
&\quad + 2\alpha x^2(t) - \alpha \sum_{i=1}^{2} x^2(t-\tau_i) + \sum_{i=1}^{2} \beta_i \tanh^2 x(t) - \sum_{i=1}^{2} \beta_i \tanh^2 x(t-\sigma) \\
&= 2q(t)x(t)\tanh x(t-\sigma) - 2p(t)x^2(t) + 2q(t)\tanh x(t-\sigma) \sum_{i=1}^{2} c_i(t)x(t-\tau_i) \\
&\quad - 2p(t)x(t) \sum_{i=1}^{2} c_i(t)x(t-\tau_i) + 2\alpha x^2(t) - \alpha \sum_{i=1}^{2} x^2(t-\tau_i) \\
&\quad + \sum_{i=1}^{2} \beta_i \tanh^2 x(t) - \sum_{i=1}^{2} \beta_i \tanh^2 x(t-\sigma) \\
&= [-2p(t) + 2\alpha]x^2(t) + \sum_{i=1}^{2} \beta_i \tanh^2 x(t) - 2p(t)x(t) \sum_{i=1}^{2} c_i(t)x(t-\tau_i) \\
&\quad - \alpha \sum_{i=1}^{2} x^2(t-\tau_i) - q(t)c_1(t)[x(t-\tau_1) - \tanh x(t-\sigma)]^2
\end{aligned}$$

$$-q(t)c_2(t)[x(t-\tau_2)-\tanh x(t-\sigma)]^2+q(t)\sum_{i=1}^{2}c_i(t)x^2(t-\tau_i)$$
$$+q(t)\tanh^2 x(t-\sigma)\sum_{i=1}^{2}c_i(t)+2q(t)x(t)\tanh x(t-\sigma)$$
$$-\sum_{i=1}^{2}\beta_i\tanh^2 x(t-\sigma).$$

Subject to the assumptions of the theorem and the estimate $\tanh^2 x(t)\le x^2(t)$, it follows that

$$\begin{aligned}
\frac{dV_2(t)}{dt} &\le [-2p(t)+2\alpha+\sum_{i=1}^{2}\beta_i]x^2(t)-2p(t)x(t)\sum_{i=1}^{2}c_i(t)x(t-\tau_i) \qquad (2.7)\\
&\quad -\alpha\sum_{i=1}^{2}x^2(t-\tau_i)+q(t)\sum_{i=1}^{2}c_i(t)x^2(t-\tau_i)\\
&\quad +q(t)\tanh^2 x(t-\sigma)\sum_{i=1}^{2}c_i(t)+2q(t)x(t)\tanh x(t-\sigma)\\
&\quad -\sum_{i=1}^{2}\beta_i\tanh^2 x(t-\sigma)\\
&= [-2p(t)+2\alpha+\beta_1+\beta_2]x^2(t)\\
&\quad -\{\sum_{i=1}^{2}(\alpha-q(t)c_i(t))x^2(t-\tau_i)+2p(t)x(t)\sum_{i=1}^{2}c_i(t)x(t-\tau_i)\}\\
&\quad -\{[\sum_{i=1}^{2}\beta_i-q(t)\sum_{i=1}^{2}c_i(t)]\tanh^2 x(t-\sigma)-2q(t)x(t)\tanh x(t-\sigma)\}\\
&= [-2p(t)+2\alpha+\beta_1+\beta_2]x^2(t)-\left[\sqrt{\alpha-q(t)c_1(t)}\,x(t-\tau_1)-\frac{p(t)c_1(t)x(t)}{\sqrt{\alpha-q(t)c_1(t)}}\right]^2\\
&\quad -\left[\sqrt{\alpha-q(t)c_2(t)}\,x(t-\tau_2)-\frac{p(t)c_2(t)x(t)}{\sqrt{\alpha-q(t)c_2(t)}}\right]^2\\
&\quad +\frac{p^2(t)c_1^2(t)}{\alpha-q(t)c_1(t)}x^2(t)+\frac{p^2(t)c_2^2(t)}{\alpha-q(t)c_2(t)}x^2(t)
\end{aligned}$$

$$
\begin{aligned}
&- \left[\sqrt{\sum_{i=1}^{2} \beta_i - q(t) \sum_{i=1}^{2} c_i(t)} \ \tanh x(t-\sigma) + q(t)x(t) \left(\sqrt{\sum_{i=1}^{2} \beta_i - q(t) \sum_{i=1}^{2} c_i(t)} \right)^{-1} \right]^2 \\
&+ \frac{q^2(t)}{\sum_{i=1}^{2} \beta_i - q(t) \sum_{i=1}^{2} c_i(t)} x^2(t) \\
\leq \ & \left(-2p(t) + 2\alpha + \beta_1 + \beta_2 + \frac{p^2(t)c_1^2(t)}{\alpha - q(t)c_1(t)} + \frac{p^2(t)c_2^2(t)}{\alpha - q(t)c_2(t)} \right) x^2(t) \\
&+ \frac{q_2^2}{\sum_{i=1}^{2} \beta_i - q(t) \sum_{i=1}^{2} c_i(t)} x^2(t) \\
\leq \ & \left[-2p_1 + 2\alpha + \beta_1 + \beta_2 + \frac{p_2^2 c_2}{\alpha - q_2 c_2} + \frac{p_2^2 c_3}{\alpha - q_2 c_3} + q_2^2 \left(\sum_{i=1}^{2} \beta_i - q_1 \sum_{i=1}^{2} c_{i+1} \right)^{-1} \right] x^2(t)
\end{aligned}
$$

for all $t \geq T \geq t_0$.

Integrating both sides of (2.7) from T to t, we obtain

$$
\begin{aligned}
V_2(t) \quad &+ \quad [2p_1 - \{2\alpha + \sum_{i=1}^{2} \beta_i + \frac{p_2^2 c_2}{\alpha - q_2 c_2} + \frac{p_2^2 c_3}{\alpha - q_2 c_3} \\
&+ \quad q_2^2 \left(\sum_{i=1}^{2} \beta_i - q_2 \sum_{i=1}^{2} c_{i+1} \right)^{-1} \}] \int_T^t x^2(s) ds \\
&\leq \quad V_2(T) < \infty.
\end{aligned}
$$

Hence, we conclude that $V_2(t)$ is bounded on $[T, \infty)$ and $x(t) \in L^2[T, \infty)$. The rest of proof is similar to that of Theorem 1 in Agarwal and Grace [1]. Therefore, we omit the details. □

References

[1] R. P. Agarwal and S. R. Grace, Asymptotic stability of certain neutral differential equations, *Math. Comput. Model.* **31** (8-9) (2000), 9–15.

[2] V. Charitonov and D. Melechor-Aguilar, On delay-dependent stability conditions. *Syst. Control Lett.* **40 (1)** (2002), 71–76.

[3] H. A. El-Morshedy and K. Gopalsamy, Nonoscillation, oscillation and convergence of a class of neutral equations, *Nonlinear Anal.: Theory Methods Appl.* **40(1-8)** (2000), 173–183.

[4] E. Fridman, New Lyapunov-Krasovskii functionals for stability of linear retarded and neutral type systems. *Syst. Control Lett.* **43** (4) (2001), 309–319.

[5] E. Fridman, Stability of linear descriptor systems with delays: a Lyapunov-based approach. *J. Math. Anal. Appl.* **273** (1) (2002), 24–44.

[6] E. Fridman and U. Shaked , H∞-control of linear state-delay descriptor systems: an LMI approach. *Linear Algebra Appl.* 351-352 (2002), 271–302.

[7] K. Gopalsamy , *Stability and Oscillation in Delay Differential Equations of Population Dynamics*. Kluwer Academic, Netherlands, 1992.

[8] K. Gopalsamy and L. Leung , Convergence under dynamical thresholds with delays. IEEE Trans. Neural Networks **8** (2) (1997), 341–348.

[9] K. Gu , *An integral inequality in the stability problem of time-delay system systems.* In: Proceedings of 39th IEEE CDC, Sydney, Australia, 2000, 2805-2810.

[10] I. Gyori and G. Ladas , *Oscillation theory of delay differential equations*. With applications. Oxford Mathematical Monographs. Oxford Science Publications. The Clarendon Press, Oxford University Press, New York, 1991.

[11] J. Hale , *Theory of functional differential equations.* Second edition. Applied Mathematical Sciences, Vol. 3. Springer-Verlag, New York-Heidelberg, 1977.

[12] J. Hale and S. M. Verduyn Lunel , *Introduction to Functional Differential Equations.* Springer-Verlag, New York, 1993.

[13] V. Kolamnovskii and A. Myshkis , Applied Theory of Functional Differential Equations. Kluwer, Boston, MA (1999).

[14] V. Kolmanovskii and V. Nosov , *Stability of Functional Differential Equations.* Academic, New York, 1986.

[15] X. F. Liao and K. W. Wong , Global exponential stability for a class of retarded functional differential equations with applications in neural networks. *J. Math. Anal. Appl.* **293** (1) (2004), 125–148.

[16] H. Logemann and S. Townley , The effects of small delays in the feedback loop on the stability of the neutral systems. *Syst. Control Lett.* **27** (5) (1996), 267–274.

[17] P. T. Nam and V. N. Phat , An improved stability criterion for a class of neutral differential equations. *Appl. Math. Lett.* **22** (2009), no. 1, 31–35.

[18] P. Park , A delay-dependent stability criterion for systems with uncertain time-invariant delays, *IEEE Trans. Automat. Contr.* **44** (4) (1999), 876–877.

[19] J. H. Park, Delay-dependent criterion for asymptotic stability of a class of neutral equations. *Appl. Math. Lett.* **17** (10) (2004), 1203–1206.

[20] D. Shaojiang Deng, L. Xiaofeng and G. Songtao , Asymptotic stability analysis of certain neutral differential equations: A descriptor system approach. *Mathematics and Computers in Simulation* **79** (10), (2009), 2981–2993.

[21] Y. G. Sun and L. Wang , Note on asymptotic stability of a class of neutral differential equations. *Appl. Math. Lett.* **19** (9) (2006), 949–953.

[22] C. Tunç and A. Sirma, Stability analysis of a class of generalized neutral equations. *J. Comput. Anal. Appl.* **12** (2010), no. 4, 754–759.

Received: 18.27.2011,

Accepted: 05.08.2012.

Department of Mathematics, Faculty of Sciences,
Yüzüncü Yıl University, 65080, Van-Turkey.
E-mail: `cemtunc@yahoo.com`