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# A tribute to Professor Nicolae Popescu (1937–2010)

by
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"Mathematics is the fundamental science of human knowledge. It is as complex and limitless as the limitless universe it explores." (Nicolae Popescu)

Nicolae Popescu was a great Romanian mathematician, creator of a powerful school of Algebra and Number Theory. Member of the Romanian Academy, and author of more than a hundred original research papers, he received wide international recognition. He obtained exceptional results which have impacted many domains of mathematics, including Category Theory, Theory of Rings and Modules, Algebraic Geometry, Galois Theory, and Number Theory.

Born in Comanda - Strehaia, Mehedinţi County, Romania, he attended high school in Craiova, then graduated in 1964 and received his Ph.D. in mathematics in 1967 from the University of Bucharest. As a very young talented researcher, he joined the Institute of Mathematics of the Romanian Academy in 1964.

He initiated in 1960 a scientific seminar for students, young researchers, and even assistant professors, and shared with them his ideas. With his vision on how to study and create mathematics, revealing its secrets with generosity, he generated emulation and led his scientific seminar, which today bears his name, for 50 years. During the communist regime, he was not allowed to travel abroad to honor important invitations from universities from more than 20 countries. This was a big disservice to him and to Romanian mathematics. Only after 1990 he was able to honor such invitations, giving talks at numerous universities from France, Switzerland, Italy, Belgium, Germany, Russia, USA, Spain, Japan, Kuwait, Mexico, Egypt, Pakistan, Chile, Nicaragua etc.

He left a deep trail not only in mathematics; through his personal example, and through the seeds he planted in those who knew him, he also left us with a wonderful spiritual work of unique freshness, beauty, and depth.

While still a student, he focused on the theory of categories. He first approached general elements of the theory of categories, homological algebra and algebraic topology, then the theory of Abelian categories. He was one of the main promoters of this theory in Romania, as evidenced by his first works and his remarkable monographs, known and appreciated internationally, which increased his widely recognized reputation throughout the mathematical community.

After D.A. Buchsbaum defined Abelian categories in 1955, A. Grothendieck published in 1957 the fundamental work *Sur quelques points d'algèbre homologique*. Its results were used by

Pierre Gabriel, who, in his 1962 thesis entitled *Des catégories abéliennes*, develops the concept of localization, showing that a quotient category of modules via a localized subcategory satisfies condition Ab5 and has many generators. Using the technique of localization, Professor Nicolae Popescu succeeded in 1964 to prove in [7] (with Pierre Gabriel) an inverse result: *any Grothendieck category (an Ab5 category) is canonically equivalent to the quotient category of modules by a localized subcategory.* This result is known as Popesco–Gabriel (Gabriel–Popescu) Theorem and has been regarded as a basic theorem in category theory; a complete proof was given by Professor Popescu in the monograph [C1]. The extensive citation of this result made Nicolae Popescu's name well known to the international mathematical community. Popesco–Gabriel Theorem was used by numerous authors, for example in Artin-Grothendieck-Verdier seminar from IEHS, 1963/1964, dedicated to the general technique of localization and its applications in algebraic geometry and in almost all works that involve the technique of localization of the Lyon School Algebra, including many doctoral theses. Among those that have obtained new results are K. Morita, who used it (1970) to prove new properties on the categories of modules and on the duality theory of modules. M. Harada (1968) gave a new proof and used it in the study of some particular Grothendieck categories. New proofs were also given, among others, by M. Takeuchi, R. Ouzilou, P.M. Cohn and Joachim Lambek. It is cited in the monographs of C. Ehresmann, *Categories et Structures*, Dunod, Paris, 1965; H. Schubert, *Kategorien. I, II*, Springer-Verlag, Berlin-New York, 1970; C. Faith, *Rings, Modules and Categories*, Springer Verlag, 1973; B. Stenström, *Rings of Quotients*, Springer Verlag, 1975, and in numerous other papers up to now. Paper [10], devoted to the general theory of localization, goes through almost all techniques known at the time. The main result is related to Grothendieck topologies defined with the aid of certain systems of morphisms, which allows identifying that category of sheaves with a category of presheaves over a certain category of fractions. In [13]–[16], [18], [21] he presented, in an original manner, the basic concepts from the theory of sheaves and categories, and some new results of himself. These papers, in fact a treatise on the general theory of categories, are cited in the monograph *Theory of Categories* by Horst Herrlich, in papers of J.E. Roos and by many others. The topics addressed in [12] have been studied also by other researchers, especially those in the school of algebra from Orsay. Paper [19] is used in works of B. Stenström, J.E. Roos, A. Hudry and others. In [20] Professor Nicolae Popescu introduced a new concept of decomposition theory, more general than that of Riley, and showed that the decomposition theories of Riley are specific to local noetherian categories. It is cited by C. Năstăsescu in his monograph *Rings, Modules, Categories*, Editura Academiei, 1976, by J. Golan in the preface of his monograph *Decomposition and dimension theory in module categories*, M. Dekker, Inc. 1977. Paper [24] presents original results on the localized systems on a commutative ring and gives a characterization of flat epimorphisms of rings, showing for the first time their close connection with the theory of localization. In [23] and [25] it is obtained an intrinsic characterization theorem of flat epimorphisms of rings, showing their close relation to localization. Paper [26] shows that, in fact, the concept of flat (left) bimorfism of rings (or quasi-order, a notion introduced by Professor Nicolae Popescu) is a natural generalization of the classical concept of order. He obtained the converse of a theorem given in 1967 by F.L. Sandomierski, and gave the answer to a problem posed in 1967 by P. Jans. In [22] semi-artinian rings are defined, and a characterization of left semi-artinian rings in terms of the Jacobson radical is given. A. Hudry has presented it at the Algebra Seminar of Lyon in 1970. This work

formed the basis of a few doctoral theses in France (Lyon and Paris) and in Romania. The analogue of prime ideal for a noncommutative ring, defined using the technique of localization, was introduced by Oscar Goldman in 1969. In [28] Professor Nicolae Popescu introduces the concept of left prime ideal of a ring, a definition which is equivalent to that of Goldman. The novelty is the concept of left super-prime ideal. It is shown that these ideals are the adequate extensions from the commutative case.

Since 1970 he turned his attention to applications of categories. Initially his works used techniques from category theory in the study of rings, and later he developed new techniques specific to the theory of rings. We mention here papers [32], [34], [36], and [41]. In [41] Professor Nicolae Popescu introduced an interesting class of Prüfer rings satisfying a usual property of Dedekind rings, reason for which he called them generalized Dedekind domains. The paper also generalized a result of Gilmer for almost-Dedekind domains to the case of generalized Dedekind domains. He succeeded to characterize the generalized-Dedekind domains in [55], [59], [70]. In [27] the rings with defined Krull-Gabriel dimension are characterized from the point of view of localized systems. It contains many results on K.G.-rings. These rings have a primary decomposition theory, a remarkable property, similar to that of the noetherian case. The semi-noetherian rings (so named by Professor Nicolae Popescu) then entered the specific literature. These works were cited by B. Stentström, J. Golan, J. Raynaud, J.A. Beachy and so on. Professor Nicolae Popescu characterized the C.P.-rings, and the f-rings which are also C.P.-rings ([29]) and generalized the theorems of classical primary decomposition in the noetherian case ([30]). In 1983 he published a paper with a topic from fields and algebraic numbers ([40]). Professor Nicolae Popescu published a first Romanian introduction to Class Field Theory ([44],[46]). Other works concerned Dedekind domains in infinite algebraic extensions ([43]), and classes of subfields of $K(X)$ ([45]), [48]).

The work of Professor Nicolae Popescu in Valuation Theory and Number Theory has a strong impact on current research. In paper [47] he gives a proof to Nagata conjecture, independent of Ohm's proof. His new method opened the path for further development of the subject. A systematic study of r.t. extensions of valuations from $K$ to $K[X]$ is made in [49] and [50]. Corollaries 2.5 and 2.6 in [50] solve three open problems of J. Ohm: *Let $v$ be a valuation on a field $K$, with residue field $k_v$, and let $w$ be a residual transcendental extension of $v$ to $K(X)$. Then the equality* $\deg(w/v) = f(w/v)e(w/v)$ *holds true if: a) $v$ is Henselian and char $k_v = 0$, or b) $v$ is of rank one, and char $k_v = 0$, or c) $v$ is of rank one and discrete.* This work opened many directions of research, see [51]–[54], [56], [58], [63]. Combining his experience in valuation theory with his expertise in localization theory, he obtained in joint work [61], [64] remarkable new results in the theory of Prüfer rings. We mention the monograph *Prüfer domains*, Marcel Dekker, 1997, by M. Fontana, J.A. Huckaba, and I.J. Papick, where Chapter II is devoted to the study of a class of Prüfer rings introduced by Professor Nicolae Popescu in [41] and [55], and contains exclusively the results of these papers, and [64]. In [106] it is given a description of all localized systems of a Prüfer domain using the characterization of a localized system in a valuation ring. Local fields, their algebraic closures, and completions of their algebraic closures, were studied in [57], [65]–[67]. Paper [60] provides a useful description of irreducible polynomials over a local field. Professor Nicolae Popescu and his collaborators introduced a number of invariants associated to some elements in the completion $\mathbb{C}_p$ of a fixed algebraic closure $\overline{\mathbb{Q}}_p$ of $\mathbb{Q}_p$. One of them is the trace series associated to an element of $\mathbb{C}_p$ (see [72]),

which represents a Krasner analytic function that encodes a significant amount of algebraic and analytic data. Some properties of trace on $\mathbb{C}_p$ are established in Theorem 1 from [98]: *Let $T$ be an element of $\mathbb{C}_p$ and let $K$ be an intermediate complete subfield between $\mathbb{Q}_p$ and $\mathbb{C}_p$ such that $Tr_{\mathbb{C}_p/K}(T)$ exists. Then for any finite extension $K \subseteq L$, $Tr_{\mathbb{C}_p/L}(T)$ exists. Moreover if $K_1 \subseteq K_2$ are finite extensions of $K$ then one has $Tr_{\mathbb{C}_p/K_1}(T) = Tr_{K_2/K_1}(Tr_{\mathbb{C}_p/K_2}(T))$*. Further investigations led to various results, such as an analog of a theorem of Barsky and Vishik, a new proof and interpretation of a theorem of Serre related to Iwasawa algebra, a Weierstrass type representation theorem, and some results of Picard type, see [94], [96], [99], [104], [107], [108], [111].

Professor Nicolae Popescu introduced in 1999 a notion of spectral norm on a fixed algebraic closure $\overline{\mathbb{Q}}$ of $\mathbb{Q}$. The spectral norm of an $x$ in $\overline{\mathbb{Q}}$ is $\|x\| = \max\{|\sigma(x)| : \sigma \in G\}$, where $G = Gal(\overline{\mathbb{Q}}/\mathbb{Q})$. As described in [88], the completion of the normed field $\left(\overline{\mathbb{Q}}, \|.\|\right)$ is a normed ring $\widetilde{\overline{\mathbb{Q}}}$ having interesting properties. Let $C(G)$ denote the $\mathbb{C}-$Banach algebra of all continuous functions defined on the absolute Galois group $G$. Then Corollary 1.11 in [88] states the following: *There is a natural $\mathbb{C}-$Banach algebra isomorphism between $\mathbb{Q}[i]$ and $C(G)$, which gives the structure of all the continuous functions defined on the absolute Galois group in language of algebraic numbers.* A Galois theory relative to some classes of closed subalgebras, the existence of topologically generating elements for closed subfields, and a Krasner type lemma were established in [73]. Professor Nicolae Popescu and his collaborators tried to extend the usual arithmetic of algebraic elements to some classes of continuous functions defined on $G$ [84], [85], [93]. By employing spectral norms, orbits and pseudoorbits, in papers [83] and [87] a special class of compact subsets of $\mathbb{C}$ and a Galois type action of $G$ on such sets are described. By starting with a $p$-adic valuation on $\mathbb{Q}$ a similar construction is described in [75], [91].

Professor Nicolae Popescu continued the study of the Nagata transform in the context of a localizing system on a ring ([76]) and the Kaplansky ideal transform ([78]). Papers [79], [81], [82], [95] are devoted to noncommutative valuations. A new result on minimal pairs for valuations on $K(X)$ was obtained in [80]. The study of metric invariants was continued in [77], [86], and [89]. Results on affine subdomains in some Tate algebras were obtained in [92], and in [101], [103] a description of non-Archimedean norms on the ring $K[x_1, ..., x_n]$ was obtained.

Professor Nicolae Popescu wrote 5 books cited in hundreds of articles and dozens of monographs. [B1] is a consistent introduction to algebraic number theory, using modern language and up to date literature in the field. [B2] (with A. Radu) is a monograph on the general theory of categories and sheaves, contributing to the development of algebra in Romania. The monographs [B3] and [B4] are devoted to Abelian categories, containing many original results. [B3] was awarded the Simion Stoilow prize by the Romanian Academy. [B4] contains applications of Abelian categories in the theory of rings and modules. [B4] is indicated in the vast *Encyclopaedia of Mathematics* as a fundamental work in abelian categories. Monograph [B5] (with L. Popescu) contains the basic results from the theory of categories, bringing the most important results up to date.

Professor Nicolae Popescu's work continues to have a major impact in current mathematical research. Many mathematicians continue to use numerous ideas from his work. We mentioned only a very small number of those who have applied his works in one form or another. All these works show a wellspring of ideas, and a profound transformations of the areas approached, with continuous and full of dedication, by this great Master.

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