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Groups with (m_1, m_2, n)-permutational property

by
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Abstract

Let G be a group, $\mathcal{A}$ and $\mathcal{B}$ be two sets of n-tuples of elements of G with $|\mathcal{A}| = m_1$ and $|\mathcal{B}| = m_2$, respectively. G is said to have the (m_1, m_2, n)-permutational property with respect to $\mathcal{A}$ and $\mathcal{B}$ if for all elements $g_1, g_2, \cdots, g_n \in G$, there exist $a_1, a_2, \cdots, a_n \in \mathcal{A}$, $b_1, b_2, \cdots, b_n \in \mathcal{B}$ and a nonidentity permutation $\sigma \in Sym_n$ such that

$$a_1 g_1 b_1 a_2 g_2 b_2 \cdots a_n g_n b_n = a_{\sigma(1)} g_{\sigma(1)} b_{\sigma(1)} a_{\sigma(2)} g_{\sigma(2)} b_{\sigma(2)} \cdots a_{\sigma(n)} g_{\sigma(n)} b_{\sigma(n)}.$$

We show that if G is (m_1, m_2, n)-permutational, then G has a characteristic subgroup N such that $|G : N|$ and $|N'|$ are both finite and have sizes bounded by functions of m_1, m_2 and n. As a consequence, if Δ is the finite conjugate center of the group, then $|G : \Delta|$ and Δ' are both finite with $|G : \Delta|$ bounded by a function of m_1, m_2 and n.

Key Words: Group, permutational property, finite conjugate center.
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1 Introduction

Throughout the paper, we let G be a multiplicative group. For $g \in G$, denote by $C_G(g)$ the centralizer of g in G, i.e., the subgroup consisting of all elements x such that $xg = gx$. Define the finite conjugate center of G as

$$\Delta = \Delta(G) = \{g \in G : |G : C_G(g)| < \infty\},$$

which is a characteristic subgroup of G. The n-permutational property P_n is introduced in [8]. Here G is said to have P_n if for all $g_1, g_2, \cdots, g_n \in G$, there exists a nonidentity permutation $\sigma \in Sym_n$ such that

$$g_1 g_2 \cdots g_n = g_{\sigma(1)} g_{\sigma(2)} \cdots g_{\sigma(n)}.$$

$Sym_n = Sym\{1, 2, \cdots, n\}$ is the symmetric group of order n. Clearly, P_2 is commutativity. It is shown [8] that if G has P_n, then $|G : \Delta|$ is finite and has size bounded by a function of n. In addition, the commutator subgroup Δ' of Δ is finite, but its order can not be bounded by a function of n.

Groups with n-permutational property have been extensively studied and various generalizations of P_n have been considered in the literature; see e.g. [4, 5, 10, 13, 11, 7]. The n-rewritable property Q_n, for example, is introduced in [4]. A group G is said to have Q_n if for all $g_1, g_2, \cdots, g_n \in G$, there exist two distinct permutations $\sigma, \tau \in Sym_n$ such that

$$g_{\sigma(1)}g_{\sigma(2)}\cdots g_{\sigma(n)} = g_{\tau(1)}g_{\tau(2)}\cdots g_{\tau(n)}.$$

Recently, the permutational property has been revisited [9, 1, 6, 14, 2] and an intriguing generalization of P_n is offered in [9]. For positive integers m and n, let $\mathcal{A}$ be a set of n-tuples of elements of G with $|\mathcal{A}| = m$. G is said to be (m,n)-permutational with respect to $\mathcal{A}$ if for all elements $g_1, g_2, \cdots, g_n \in G$, there exist $a_1, a_2, \cdots, a_n \in \mathcal{A}$ and a nonidentity permutation $\sigma \in Sym_n$ such that

$$g_1a_1g_2a_2\cdots g_na_n = g_{\sigma(1)}a_{\sigma(1)}g_{\sigma(2)}a_{\sigma(2)}\cdots g_{\sigma(n)}a_{\sigma(n)}.$$

Drawing on the techniques from [13], it is shown [9, Proposition 4.1] that if G is (m,n)-permutational with respect to $\mathcal{A}$, then G has a characteristic subgroup N such that $|G:N|$ and $|N'|$ are finite and both have sizes bounded by functions of m and n (Unfortunately, there is a typo in the statement of [9, Proposition 4.1 (ii)]).

In this paper, we move a further step beyond (m,n)-permutation by considering another interesting generalization of P_n: (m_1,m_2,n)-permutational property. We have the following definition.

Definition 1. *Let m_1, m_2, n be positive integers and suppose that $\mathcal{A}$ and $\mathcal{B}$ are two sets of n-tuples of elements of G with $|\mathcal{A}| = m_1$ and $|\mathcal{B}| = m_2$, respectively. A group G is said to be (m_1,m_2,n)-permutational with respect to $\mathcal{A}$ and $\mathcal{B}$ if for every n-tuple $(g_1, g_2, \cdots, g_n)$ of elements of G there exist n-tuples $(a_1, a_2, \cdots, a_n) \in \mathcal{A}$, $(b_1, b_2, \cdots, b_n) \in \mathcal{B}$ and a nonidentity permutation $\sigma \in Sym_n$ such that*

$$a_1g_1b_1a_2g_2b_2\cdots a_ng_nb_n = a_{\sigma(1)}g_{\sigma(1)}b_{\sigma(1)}a_{\sigma(2)}g_{\sigma(2)}b_{\sigma(2)}\cdots a_{\sigma(n)}g_{\sigma(n)}b_{\sigma(n)}.$$

We show that the techniques developed in [13, 9] can yield similar results for (m_1,m_2,n)-permutational groups.

The rest of the paper is organized as follows. In Section 2, we present the main results for (m_1,m_2,n)-permutational groups. In Section 3, we provide the proofs. Finally, we give a further generalization on the (m_1,m_2,n)-permutational property in Section 4.

2 Main results

We first recall some notations introduced in [13]. Suppose that G is a group and T is a subset of G. T is said to have finite index in G if there are $g_1, g_2, \cdots, g_n \in G$ such that

$$G = Tg_1 \cup Tg_2 \cup \cdots \cup Tg_n$$

for some finite n. The index $|G:T|$ is defined to be the minimum such integer n. It is clear that if T is a subgroup of G, this definition agrees with the ordinary index of a subgroup. For an integer k, define

$$\Delta_k = \Delta_k(G) = \{g \in G : |G:C_G(g)| \le k\}.$$

The set Δ_k is a normal subset of G and $\Delta_k \cdot \Delta_l \subseteq \Delta_{kl}$ for all integers k and l. Our main result establishes as follows.

Theorem 1. *Let G be an (m_1, m_2, n)-permutational group with respect to $\mathcal{A}$ and $\mathcal{B}$. Set $k = m_1 \cdot m_2 \cdot n!$. Then we have*

(i) $|G : \Delta_k| \leq k \cdot (k+1)!$, and

(ii) G has a characteristic subgroup N with $|G : N| \leq k \cdot (k+1)!$, and with $|N'|$ finite and bounded by a function of m_1, m_2 and n.

Recall that Δ is the finite conjugate center of G. As a consequence, we have the following corollary.

Corollary 1. *Let G be an (m_1, m_2, n)-permutational group with respect to $\mathcal{A}$ and $\mathcal{B}$. Set $k = m_1 \cdot m_2 \cdot n!$. Then $|G : \Delta| \leq k \cdot (k+1)!$ and $|\Delta'|$ is finite.*

A group G is said to be perfect if $G = G'$. Following [9], we say G is normally perfect if all normal subgroups of G are perfect. Clearly, any nonabelian simple group is perfect. The following corollary can be viewed as a generalization of [5, Theorem 1] and [9, Corollary 2.8].

Corollary 2. *Let G be a normally perfect group satisfying the (m_1, m_2, n)-permutational property. Then G has finite order bounded by a function of m_1, m_2 and n.*

3 Proofs

In this section, we provide the proofs of the aforementioned results. We will capitalize on the techniques in [13, 9] and the following several lemmas are useful.

Lemma 1. [13] *Let $S = \cup_{i=1}^{k} H_i g_i$ be a finite union of cosets of the subgroups H_i of G and assume that $S \neq G$. Then there exist $x_1, x_2, \cdots, x_l \in G$, with $l = (k+1)!$ such that $\cap_{i=1}^{l} S x_i = \emptyset$. In particular, if T is a subset of G with $G = S \cup T$, then $|G : T| \leq (k+1)!$*

Lemma 2. [13, 12] *Let $S = \cup_{i=1}^{k} H_i g_i$ be a finite union of cosets of subgroups H_i of G. If $|G : H_i| > k$ for every $1 \leq i \leq k$, then $S \neq G$.*

Lemma 3. [9] *Let k and l be positive integers and assume that $|G : \Delta_k| \leq l$. If N is the subgroup of G generated by Δ_k, then N is a characteristic subgroup of G with $|G : N| \leq l$, and with $|N'|$ finite and bounded by a function of k and l.*

Following [13], we define a linear monomial in the noncommuting variables $\xi_1, \xi_2, \cdots, x_n$ to be a monic monomial μ of the form $\xi_{i_1} \xi_{i_2} \cdots \xi_{i_t}$ with all i_j distinct and $t = \deg \mu$. We have $\mu = 1$ if and only if $\deg \mu = 0$. Furthermore, it is straightforward to check that $(n+1)!$ is a (quite loose) upper bound of the number of linear monomials in n variables.

Proof of Theorem 1. We assume by way of contradiction that $|G : \Delta_k| > k \cdot (k+1)!$. Let $M_1 = \emptyset$ and, for $j \geq 2$, let M_j denote the set of all linear monomials in the noncommuting variables $\xi_j, \xi_{j+1}, \cdots, \xi_n$. According to the above comments we have $|M_j| \leq n!$.

In what follows, we first show by induction on $j = 1, 2, \cdots, n$ that, for any $g_j, g_{j+1}, \cdots, g_n \in G$, there exist $(a_1, a_2, \cdots, a_n) \in \mathcal{A}$ and $(b_1, b_2, \cdots, b_n) \in \mathcal{B}$ such that either

$$a_1 g_1 b_1 a_2 g_2 b_2 \cdots a_n g_n b_n = a_{\sigma(1)} g_{\sigma(1)} b_{\sigma(1)} a_{\sigma(2)} g_{\sigma(2)} b_{\sigma(2)} \cdots a_{\sigma(n)} g_{\sigma(n)} b_{\sigma(n)}$$

for some $1 \neq \sigma \in Sym\{j, j+1, \cdots, n\}$ or $\mu(a_j g_j b_j, a_{j+1} g_{j+1} b_{j+1}, \cdots, a_n g_n b_n) \in \Delta_k$ for some monomial $\mu \in M_j$.

Since G is an (m_1, m_2, n)-permutational group with respect to $\mathcal{A}$ and $\mathcal{B}$, the result for $j = 1$ holds by definition. Suppose the result holds for some $j < n$. Fix $g_{j+1}, g_{j+2}, \cdots, g_n \in G$ and let g play the role of the jth variable. Let $\mu \in M_{j+1}$. If $\mu(a_{j+1} g_{j+1} b_{j+1}, \cdots, a_n g_n b_n) \in \Delta_k$ for some $(a_1, a_2, \cdots, a_n) \in \mathcal{A}$ and $(b_1, b_2, \cdots, b_n) \in \mathcal{B}$, then we are done. Hence, we may assume that $\mu(a_{j+1} g_{j+1} b_{j+1}, \cdots, a_n g_n b_n) \notin \Delta_k$ for every $\mu \in M_{j+1}$, for every $(a_1, a_2, \cdots, a_n) \in \mathcal{A}$ and $(b_1, b_2, \cdots, b_n) \in \mathcal{B}$.

Next, for each $1 \neq \sigma \in Sym\{j, j+1, \cdots, n\}$, $\alpha = (a_1, a_2, \cdots, a_n) \in \mathcal{A}$ and $\beta = (b_1, b_2, \cdots, b_n) \in \mathcal{B}$, set

$$S_{\sigma,\alpha,\beta} = \{g = g_j \in G : a_j g_j b_j a_{j+1} g_{j+1} b_{j+1} \cdots a_n g_n b_n = a_{\sigma(j)} g_{\sigma(j)} b_{\sigma(j)} \cdot a_{\sigma(j+1)} g_{\sigma(j+1)} b_{\sigma(j+1)} \cdots a_{\sigma(n)} g_{\sigma(n)} b_{\sigma(n)}\}.$$

If $S_{\sigma,\alpha,\beta} \neq \emptyset$ and σ fixes j, then we can cancel the beginning $a_j g_j b_j$ factors and conclude that

$$a_{j+1} g_{j+1} b_{j+1} \cdots a_n g_n b_n = a_{\sigma(j+1)} g_{\sigma(j+1)} b_{\sigma(j+1)} \cdots a_{\sigma(n)} g_{\sigma(n)} b_{\sigma(n)}$$

for some $1 \neq \sigma \in Sym\{j+1, \cdots, n\}$. Hence, we can assume that if $S_{\sigma,\alpha,\beta} \neq \emptyset$, then σ does not fix j.

Now suppose that $S_{\sigma,\alpha,\beta} \neq \emptyset$ and let $g \in S_{\sigma,\alpha,\beta}$ so that

$$a_j g b_j a_{j+1} g_{j+1} b_{j+1} \cdots a_n g_n b_n = a_{\sigma(j)} g_{\sigma(j)} b_{\sigma(j)} a_{\sigma(j+1)} g_{\sigma(j+1)} b_{\sigma(j+1)} \cdots a_{\sigma(n)} g_{\sigma(n)} b_{\sigma(n)}.$$

If we set $\rho = a_{j+1} g_{j+1} b_{j+1} \cdots a_n g_n b_n$, then we obtain

$$\begin{aligned} \rho &= a_{j+1} g_{j+1} b_{j+1} \cdots a_n g_n b_n \\ &= (a_j g b_j)^{-1} (a_{\sigma(j)} g_{\sigma(j)} b_{\sigma(j)} a_{\sigma(j+1)} g_{\sigma(j+1)} b_{\sigma(j+1)} \cdots)(a_j g b_j) \cdots a_{\sigma(n)} g_{\sigma(n)} b_{\sigma(n)} \\ &= (a_j g b_j)^{-1} \lambda_{\sigma,\alpha,\beta} (a_j g b_j) \bar{\lambda}_{\sigma,\alpha,\beta}, \end{aligned}$$

where $\lambda_{\sigma,\alpha,\beta}$ and $\bar{\lambda}_{\sigma,\alpha,\beta}$ depend only on σ, α and β. Indeed, since $\sigma(j) \neq j$, $\lambda_{\sigma,\alpha,\beta}$ is a linear monomial in M_{j+1} evaluated at $a_{j+1} g_{j+1} b_{j+1}, \cdots, a_n g_n b_n$, and therefore, we have $\lambda_{\sigma,\alpha,\beta} \notin \Delta_k$ by assumption. Note that the above equation is equivalent to

$$g^{-1} a_j^{-1} \lambda_{\sigma,\alpha,\beta} a_j g = b_j \rho (\bar{\lambda}_{\sigma,\alpha,\beta})^{-1} b_j^{-1}.$$

It follows that $S_{\sigma,\alpha,\beta}$ consists of precisely one right coset of $C_G(a_j^{-1} \lambda_{\sigma,\alpha,\beta} a_j)$, say $S_{\sigma,\alpha,\beta} = C_G(a_j^{-1} \lambda_{\sigma,\alpha,\beta} a_j) h_{\sigma,\alpha,\beta}$. Write

$$S = \cup_{\sigma,\alpha,\beta} S_{\sigma,\alpha,\beta} = \cup_{\sigma,\alpha,\beta} C_G(a_j^{-1} \lambda_{\sigma,\alpha,\beta} a_j) h_{\sigma,\alpha,\beta}.$$

Since $\lambda_{\sigma,\alpha,\beta} \notin \Delta_k$, it implies that $a_j^{-1} \lambda_{\sigma,\alpha,\beta} a_j \notin \Delta_k$ and $|G : C_G(a_j^{-1} \lambda_{\sigma,\alpha,\beta} a_j)| > k$. Since there are at most $m_1 \cdot m_2 \cdot n! = k$ cosets in the above union for S, we conclude from Lemma 2 that $S \neq G$. Consequently, by virtue of Lemma 1, we obtain that $G \backslash S$ has index $\leq (k+1)!$ in G.

Finally, set $M_j \backslash M_{j+1} = F_j$ and let $\mu \in F_j$ so that μ involves the variable ξ_j. Thus we can write $\mu = \mu' \xi_j \mu''$, where μ' and μ'' are linear monomials in the variables $\xi_{j+1}, \xi_{j+2}, \cdots, \xi_n$. If

$\alpha = (a_1, a_2, \cdots, a_n) \in \mathcal{A}$ and $\beta = (b_1, b_2, \cdots, b_n) \in \mathcal{B}$, then $\mu(a_j g_j b_j, \cdots, a_n g_n b_n) \in \Delta_k$ if and only if

$$\begin{aligned} a_j g b_j = a_j g_j b_j &\in \mu'(a_{j+1} g_{j+1} b_{j+1}, \cdots, a_n g_n b_n)^{-1} \Delta_k \mu''(a_{j+1} g_{j+1} b_{j+1}, \cdots, a_n g_n b_n)^{-1} \\ &= \Delta_k g_{\mu,\alpha,\beta}, \end{aligned}$$

since Δ_k is a normal subset of G. In particular, this occurs if and only if $g \in \Delta_k a_j^{-1} g_{\mu,\alpha,\beta} b_j^{-1}$, a fixed right translate of Δ_k. Hence, if $T = \cup_{\mu,\alpha,\beta} \Delta_k a_j^{-1} g_{\mu,\alpha,\beta} b_j^{-1}$, where the union is over all $\mu \in F_j$, $\alpha \in \mathcal{A}$ and $\beta \in \mathcal{B}$, then the inductive assumption implies that $G = S \cup T$. In fact, suppose $x \in G$. If there exist σ, α and β with

$$a_j g_j b_j \cdots a_n g_n b_n = a_{\sigma(j)} g_{\sigma(j)} b_{\sigma(j)} \cdots a_{\sigma(n)} g_{\sigma(n)} b_{\sigma(n)}$$

and $g_j = x$, then $x \in S_{\sigma,\alpha,\beta} \subseteq S$. On the other hand, if there exist μ, α and β with

$$\mu(a_j g_j b_j, \cdots, a_n g_n b_n) \in \Delta_k$$

and $g_j = x$, then $x \in \Delta_k a_j^{-1} g_{\mu,\alpha,\beta} b_j^{-1} \subseteq T$.

It follows that $T \supseteq G \backslash S$, so

$$|G : T| \leq |G : G \backslash S| \leq (k+1)!.$$

But T is a union of at most $|\mathcal{A}| \cdot |\mathcal{B}| \cdot |F_j| \leq m_1 \cdot m_2 \cdot |M_j| \leq m_1 \cdot m_2 \cdot n! = k$ right translates of Δ_k, so we see that

$$|G : \Delta_k| \leq k \cdot |G : T| \leq k \cdot (k+1)!$$

a contradiction by assumption. Hence, the inductive statement is proved.

In particular, the inductive result holds when $j = n$. Here, there are no nonidentity permutations in $Sym\{n\}$, and $M_n = \{\xi_n\}$. We conclude that, for each $g \in G$, there exist $\alpha \in \mathcal{A}$ and $\beta \in \mathcal{B}$ with $a_n g b_n \in \Delta_k$ and hence with $g \in \Delta_k a_n^{-1} b_n^{-1}$. In other words, we have $G = \cup_{\alpha,\beta} \Delta_k a_n^{-1} b_n^{-1}$, where a_n and b_n are the nth entries of α and β, respectively. Thus, $|G : \Delta_k| \leq m_1 \cdot m_2 \leq k$. Hence, the assumption $|G : \Delta_k| > k \cdot (k+1)!$ is false, and part (i) of the theorem is proved.

As for part (ii), set $l = k \cdot (k+1)!$, and let N be the characteristic subgroup of G generated by Δ_k. Since $|G : \Delta_k| \leq l$, Lemma 3 readily yields the result. □

To prove Corollary 1, we need the following lemma.

Lemma 4. [13, 16] *Let G be a group and let k be a positive integer.*

(i) If $|G'| \leq k$, then $G = \Delta_k(G)$.

(ii) If $G = \Delta_k(G)$, then $|G'| \leq (k^4)^{k^4}$.

Proof of Corollary 1. It follows from Theorem 1 and the fact $\Delta_k \subseteq \Delta$ that

$$|G : \Delta| \leq |G : \Delta_k| \leq k \cdot (k+1)!.$$

Note that Δ is a subgroup of G and then we have

$$|\Delta : \Delta_k| \leq |G : \Delta_k| \leq k \cdot (k+1)!.$$

Thus, $\Delta = \cup_i \Delta_k g_i$ is a finite union of translates of Δ_k. Since every $g_i \in \Delta$ has only finitely many conjugates in G, there exists an integer l with $g_i \in \Delta_l$ for all i. Consequently, $\Delta = \Delta_k \Delta_l \subseteq \Delta_{kl}$ and thus $\Delta = \Delta_{kl}$. Using Lemma 4 (ii), we easily obtain that Δ', the commutator subgroup of Δ, is finite. □

Proof of Corollary 2. Since G is (m_1, m_2, n)-permutational with respect to $\mathcal{A}$ and $\mathcal{B}$, Theorem 1 implies that G has a normal subgroup N with both $|G : N|$ and $|N'|$ bounded by functions of m_1, m_2 and n. Hence, we have $N = N'$ since G is normally perfect. Thus $|G| = |G : N| \cdot |N'|$ is bounded by a function of m_1, m_2 and n. □

4 Discussion

Although we have stated Theorem 1 only for two sets $\mathcal{A}$ and $\mathcal{B}$ of n-tuple of elements of G, the techniques generalize to the case of an arbitrarily large but bounded number of such sets. Here, we put forward a further generalization of permutational property and list the results without proof.

Definition 2. *Let c, d, $m_1^{(1)}, \cdots, m_1^{(c)}$ and $m_2^{(1)}, \cdots, m_2^{(d)}$ be positive integers. Suppose that $\mathcal{A}^{(1)}, \cdots, \mathcal{A}^{(c)}$ and $\mathcal{B}^{(1)}, \cdots, \mathcal{B}^{(d)}$ are sets of n-tuples of elements of G with $|\mathcal{A}^{(i)}| = m_1^{(i)}$ for $1 \le i \le c$ and $|\mathcal{B}^{(j)}| = m_2^{(j)}$ for $1 \le j \le d$, respectively. A group G is said to be $\left(\{m_1^{(i)}\}_{i=1}^c, \{m_2^{(j)}\}_{j=1}^d, n\right)$-permutational with respect to $\{\mathcal{A}^{(i)}\}_{i=1}^c$ and $\{\mathcal{B}^{(j)}\}_{j=1}^d$ if for every n-tuple $(g_1, g_2, \cdots, g_n)$ of elements of G there exist n-tuples $(a_1^{(i)}, a_2^{(i)}, \cdots, a_n^{(i)}) \in \mathcal{A}^{(i)}$ for $1 \le i \le c$, $(b_1^{(j)}, b_2^{(j)}, \cdots, b_n^{(j)}) \in \mathcal{B}^{(j)}$ for $1 \le j \le d$ and a nonidentity permutation $\sigma \in Sym_n$ such that*

$$\begin{aligned} &a_1^{(1)} \cdots a_1^{(c)} g_1 b_1^{(1)} \cdots b_1^{(d)} \cdots a_n^{(1)} \cdots a_n^{(c)} g_n b_n^{(1)} \cdots b_n^{(d)} \\ &\quad = a_{\sigma(1)}^{(1)} \cdots a_{\sigma(1)}^{(c)} g_{\sigma(1)} b_{\sigma(1)}^{(1)} \cdots b_{\sigma(1)}^{(d)} \cdots a_{\sigma(n)}^{(1)} \cdots a_{\sigma(n)}^{(c)} g_{\sigma(n)} b_{\sigma(n)}^{(1)} \cdots b_{\sigma(n)}^{(d)}. \end{aligned}$$

Theorem 2. *Let G be an $\left(\{m_1^{(i)}\}_{i=1}^c, \{m_2^{(j)}\}_{j=1}^d, n\right)$-permutational group with respect to $\{\mathcal{A}^{(i)}\}_{i=1}^c$ and $\{\mathcal{B}^{(j)}\}_{j=1}^d$. Set $k = m_1^{(1)} \cdots m_1^{(c)} \cdot m_2^{(1)} \cdots m_2^{(d)} \cdot n!$. Then we have*

(i) $|G : \Delta_k| \le k \cdot (k+1)!$, and

(ii) G has a characteristic subgroup N with $|G : N| \le k \cdot (k+1)!$, and with $|N'|$ finite and bounded by a function of $\{m_1^{(i)}\}_{i=1}^c$, $\{m_2^{(j)}\}_{j=1}^d$ and n.

Corollary 3. *Let G be an $\left(\{m_1^{(i)}\}_{i=1}^c, \{m_2^{(j)}\}_{j=1}^d, n\right)$-permutational group with respect to $\{\mathcal{A}^{(i)}\}_{i=1}^c$ and $\{\mathcal{B}^{(j)}\}_{j=1}^d$. Set $k = m_1^{(1)} \cdots m_1^{(c)} \cdot m_2^{(1)} \cdots m_2^{(d)} \cdot n!$. Then $|G : \Delta| \le k \cdot (k+1)!$ and $|\Delta'|$ is finite.*

Corollary 4. *Let G be a normally perfect group satisfying the $\left(\{m_1^{(i)}\}_{i=1}^c, \{m_2^{(j)}\}_{j=1}^d, n\right)$-permutational property. Then G has finite order bounded by a function of $\{m_1^{(i)}\}_{i=1}^c$, $\{m_2^{(j)}\}_{j=1}^d$ and n.*

It is obvious that any finite group of order n has permutational property P_n. Thus, the automorphism group $Aut(\mathcal{G})$ for a finite graph $\mathcal{G}$ trivially has $P_{|Aut(\mathcal{G})|}$. The same thing is true for $\left(\{m_1^{(i)}\}_{i=1}^{c}, \{m_2^{(j)}\}_{j=1}^{d}, n\right)$-permutational property if we carefully choose the sets $\{\mathcal{A}^{(i)}\}_{i=1}^{c}$ and $\{\mathcal{B}^{(j)}\}_{j=1}^{d}$. An interesting question would be to ask the minimum non-trivial n of $\left(\{m_1^{(i)}\}_{i=1}^{c}, \{m_2^{(j)}\}_{j=1}^{d}, n\right)$-permutational property satisfied by the group $Aut(\mathcal{G})$. What if it is transitive (c.f. [3, 15])?

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