PROBLEMS FOR COMPETITIONS AND OLYMPIADS

Junior Level

C.O:5195. Let a, b be integer numbers which are not divisible by 3. Prove that the equation $x^2 - abx + a^2 + b^2 = 0$ has no integer solutions.

I. Cucurezeanu, Constanţa

C.O:5196. Find all integers $n > 3$ such that in any regular n-gon there exists at least a diagonal parallel to a side of the n-gon.

Romeo Raicu, Blaj

C.O:5197. Find all integers $m, n \geq 0$ such that $18^m + 9^n + 1$ is a square.

I. Cucurezeanu, Constanţa

C.O:5198. Show that $\frac{x+y}{z^2} + \frac{y+z}{x^2} + \frac{z+x}{y^2} \geq \frac{2}{x} + \frac{2}{y} + \frac{2}{z}$, for any real numbers $x, y, z > 0$.

Marian Cucoaneş, Mărăşeşti

Senior Level

C.O:5199. Solve the equation:

$$2^{\frac{1}{2}-\cos 2x} \cdot \sin^2 x + 2^{\frac{1}{2}+\cos 2x} \cdot \cos^2 x = \sqrt{2}.$$

Ioan Băetu, Botoşani

C.O:5200. Prove that for any rational number $a > 0$ there exist positive integers m and n such that $a = \frac{\varphi(m)}{\varphi(n)}$.

Mihai Onucu Drimbe and *Petru Mironescu*, Roman

C.O:5201. Find all differentiable functions $f : \mathbb{R} \to \mathbb{R}$ such that for each $a \in \mathbb{R}$, the graph of the restriction of f to $[a, \infty)$ lies above the tangent at $(a, f(a))$ to the graph of f, and the graph of the restriction of f to $(-\infty, a]$ lies beneath that tangent.

Dinu Şerbănescu, Bucharest

C.O:5202. Let K be a subfield of the field L, $K \neq L$, such that for any $a \in L \setminus K$, there exists $b \in L \setminus K$ with $a + b \in K$, $ab \in K$.

i) If $1 + 1 \neq 0$, prove that there exists $\lambda \in L \setminus K$ with the properties:

$$\lambda^2 \in K \text{ and } L = \{x + y\lambda \mid x, y \in K\}.$$

ii) If $1 + 1 = 0$, the conclusion of i) is true?

Marcel Ţena, Bucharest