PROBLEMS FOR COMPETITIONS AND OLYMPIADS

Junior Level

C.O:5155. Find all positive integers a given that the sets

$$A = \{2^1, 2^2, 2^3, \ldots, 2^{2010}\} \quad \text{and} \quad B = \{a, 2^1 a, 2^2 a, \ldots, 2^n a\},$$

where $n \geq 2$ is an integer, have exactly one common element.

Cosmin Manea and *Dragoş Petrică*, Piteşti

C.O:5156. Let ABC be a triangle and D, E, F be the intersection points of the incircle with the sides BC, CA, AB respectively. Suppose M, P, Q are the orthocentres of the triangles AEF, BDF, CDE, respectively. Prove that the lines DM, EP and FQ are concurrent.

Ana Maria Niţă, student, Bucharest

C.O:5157. Let p be an odd prime. Find all positive integers a and b such that $a^2 + pa = b^2$.

Cosmin Manea and *Dragoş Petrică*, Piteşti

C.O:5158. Let a and b be positive real numbers with $b < a$. Show that $8(a-b)(a^2+b^2) \leq 5a^3$ if and only if $2b > a$.

Romanţa Ghiţă and *Ioan Ghiţă*, Blaj

Senior Level

C.O:5159. Suppose A is a non-empty set. Is there a bijective function $f : A \to A$ such that there exists $H \subset A$, $H \neq \emptyset$, with $f(H) \subset H$, and $g : H \to H$, $g(x) = f(x), x \in H$ is not bijective?

Romanţa Ghiţă and *Ioan Ghiţă*, Blaj

C.O:5160. Let ABC be a triangle and let P be the intersection point of the tangents from B and C to the circumcircle K. The parallel from P to tangent in A to K intersects the lines AB and AC at M and N respectively. Prove that the ortocentre of he triangle AMN is located on the circumcircle K of ABC.

Dinu Şerbănescu, Bucharest

C.O:5161. Let a, b, c be the side lengths of a triangle and let k be a positive real number. Prove that:

$$\frac{a^k}{(b+c)^2} + \frac{b^k}{(c+a)^2} + \frac{c^k}{(a+b)^2} \geq \frac{1}{2}\left(\frac{a^k}{a^2+bc} + \frac{b^k}{b^2+ca} + \frac{c^k}{c^2+ab}\right).$$

Claudiu Mîndrilă, student, Târgovişte

C.O:5162. Let O be the circumcentre and let R be the circumradius of a triangle ABC. Let k and K be the circles tangent to the rays $(AB$ and $(AC$, as well as to the circumcircle of the triangle. Denote A_1 and A_2 the centers of k and K. Show that $(OA_1 + OA_2)^2 - A_1 A_2^2 = 4R^2$.

Neculai Roman, Mirceşti, Iaşi