PROBLEMS FOR COMPETITIONS AND OLYMPIADS

Junior Level

C.O:5027. Let a, b be positive real numbers with $a + b = 1$. Solve the equation:

$$\sqrt[4]{x + \frac{1}{a^6}} + \sqrt[4]{x + \frac{1}{b^6}} = 4\sqrt{2}.$$

Vasile Chiriac, Bacău

C.O:5028. Let $VABC$ be a tetrahedron and let O be the foot of the perpendicular from V on the plane (ABC). Define points A', B', C' as meeting points of lines AO, BO, CO with lines BC, CA, AB respectively. Given that the tetrahedra $VOAB$, $VOBC$, $VOCA$ have the same volume, prove that the parallel lines from A', B', C' to VA, VB, VC respectively are concurrent.

Gh. Stoica, Petroşani

C.O:5029. Find the sum of all two-digit integer numbers with an odd digit and an even digit.

Andrei Ion, student, Tulcea

C.O:5030. Let $A_1, A_2, \ldots, A_n$ be distinct points in plane. Prove that one can joint all points with a non self-intersecting broken line.

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Senior Level

C.O:5031. Let a, b, c, d be real numbers such that $\{a, b\} \neq \{c, d\}$. Show that the equation $\sqrt{x+a} + \sqrt{x+b} = \sqrt{x+c} + \sqrt{x+d}$ has at most one real solution.

Marius Cavachi, Constanţa

C.O:5032. Let ABC be an equilateral triangle and let D be a point on the minor arc $\overset{\frown}{BC}$ of the circumcircle. Lines BD and AC meet at point M, and lines CD and AB meet at point N. Show that $(AD$ is a simedian in the triangle AMN.

Gh. Szöllösy, Sighetu Marmaţiei

C.O:5033. Prove that $\displaystyle\sum_{k=1}^{n} \cos^m \frac{k\pi}{n} \cdot \cos \frac{mk\pi}{n} = \frac{n}{2^m}$, for all $m, n \in \mathbb{N}$, $n \geq 3$, $n \geq m + 1$.

Vasile Berghea, Avrig, Sibiu

C.O:5034. Consider $\{a_n\}_{n\geq 1}$ a sequence defined by $a_1 > 0$ and $a_{n+1} = \dfrac{1}{2}\left(a_n + \dfrac{n}{a_n}\right)$, $n \geq 1$. Show that $\lim\limits_{n\to\infty} (a_n - \sqrt{n}) = 0$.

Marius Cavachi, Constanţa