C.O:5001. Fie $a > 1$ un număr real fixat. Să se calculeze:

$$\lim_{n\to\infty} n \int_1^a \frac{\ln(1+x^n)}{x^n}\,\mathrm{d}x.$$

Cristian Chiser, Craiova

C.O:5002. Fie A un inel comutativ cu n elemente având exact n^2 funcţii polinomiale $f : A \to A$. Să se arate că $x^2 = x$, oricare ar fi $x \in A$.

Bogdan Vioreanu, Yale University, U.S.A.

PROBLEMS FOR COMPETITIONS AND OLYMPIADS

Junior Level

C.O:4995. Find the real numbers $a_1, a_2, \ldots, a_n, \ldots$ such that:

$$a_{n+1} = \sqrt{2+a_n} + \sqrt{2-a_n},$$

for all $n \geq 1$.

Maria Elena Panaitopol, Bucharest

C.O:4996. Show that there exists infinitely many integers a, b, c so that $a^2 + b^2 = c^2 + 3$.

* * *

C.O:4997. Consider 9 positive integer numbers which are divisors of 30^{2009}. Prove that one can find two of them such that their product a square.

Bogdan Vioreanu, Yale University, U.S.A.

C.O:4998. Points E and F are given on the side BC of a convex quadrilateral $ABCD$, E between B and F. If $\sphericalangle BAE = \sphericalangle CDF$ and $\sphericalangle EAF = \sphericalangle FDE$, show that $\sphericalangle FAC = \sphericalangle EDB$.

* * *

Senior Level

C.O:4999. Let $n \geq 2$ be an integer. Find the number of integers $k \in \{1, 2, \ldots, n\}$ such that k divides $5^{n!} - 3^{n!}$.

Diana Savin, Constanţa

C.O:5000. Let $f \in \mathbb{Z}[X]$ be a monic polynomial with $\deg f \geq 2$. Show that there exists infinitely many numbers $x \in \mathbb{R} \setminus \mathbb{Q}$, such that $f(x) \in \mathbb{Z}$.

Vlad Matei, Bucharest

C.O:5001. Let $a > 1$ a real number. Compute:

$$\lim_{n\to\infty} n \int_1^a \frac{\ln(1+x^n)}{x^n}\,\mathrm{d}x.$$

Cristian Chiser, Craiova

C.O:5002. Let A be a commutative ring with n elements such that there exist exactly n^2 polynomial functions $f : A \to A$. Prove that $x^2 = x$, for all $x \in A$.

Bogdan Vioreanu, Yale University, U.S.A.