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Existence and Asymptotic Behavior of Global Solutions bf for Some Nonlinear Petrovsky System

by
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Abstract

The initial-boundary value problem for some nonlinear Petrovsky system with viscoelastic term in bounded domain is studied. The existence of global solutions for this problem is proved by constructing a stable set in $H_0^2(\Omega)$ and the decay of solution energy is established by applying a difference inequality due to M.Nakao.

Key Words: Initial-boundary value problem; Nonlinear Petrovsky system; Global solutions; Viscoelastic term.
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1 Introduction

In this paper, we are concerned with the following initial-boundary value problem of nonlinear Petrovsky system

$$\begin{cases} u_{tt}+\Delta^2 u-\displaystyle\int_0^t g(t-s)\Delta^2 u(s)ds+a|u_t|^m u_t \\ \qquad\qquad = b|u|^p u, \quad (x,t)\in\Omega\times R^+, \\ u(x,0)=u_0(x)\in H_0^2(\Omega),\ u_t(x,0)=u_1(x)\in L^2(\Omega), \quad x\in\Omega, \\ u(x,t)=\dfrac{\partial u}{\partial\nu}(x,t)=0, \quad x\in\partial\Omega, t\geq 0, \end{cases} \tag{1.1}$$

where $m\geq 0,\ p>0$ and $a,\ b>0$ are real numbers, Ω is a bounded domain in R^n with smooth boundary $\partial\Omega$ so that the divergence theorem can be applied, ν is unit outward normal on $\partial\Omega$, and $\frac{\partial u}{\partial\nu}$ denotes the normal derivation of u.

A.Guesmia [1] considered the equation

$$u_{tt}+\Delta^2 u+q(x)u+f(u_t)=0, \quad x\in\Omega,\ t>0, \tag{1.2}$$

where f is a continuous and increasing function with $f(0)=0$, and $q:\Omega\longrightarrow[0,+\infty)$ is a bounded function, and he proved a global existence and a regularity result of the equation (1.2) with initial-boundary value conditions. Under suitable growth conditions on f, he also established decay results for weak and strong solutions. In addition, results similar to above system, coupled with a semi-linear wave equation, are also established by A.Guesmia [2]. As $q(x)u+f(u_t)$ in (1.2) is replaced by $\Delta^2 u_t+\Delta f(\Delta u)$, M.Aassila and A.Guesmia [3] obtained an exponential decay theorem through the use of an important lemma of V.Komornik [4]. S.A. Messaoudi [5] set up an existence result of the initial-boundary value problem of equation

$$u_{tt} + \Delta^2 u + a|u_t|^m u_t = b|u|^p u, \tag{1.3}$$

and showed that the solution continues to exist globally if $m \geq p$; however, it blows up in finite time if $m < p$. S.T.Wu and L.Y.Tsai [6] also study the problem (1.3), they showed that the solution is global in time under some conditions without the relation between m and p. They also proved the local solution blows up in finite time if $p > m$ and the initial energy is positive.

In this paper, we prove the global existence for the problem (1.1) by constructing a stable set in $H_0^2(\Omega)$ and the decay of solution energy by applying a difference inequality due to M.Nakao.

We adopt the usual notations and convention. Let $H^2(\Omega)$ denote the Sobolev space with the usual scalar products and norm. Meanwhile, $H_0^2(\Omega)$ denotes the closure in $H^2(\Omega)$ of $C_0^\infty(\Omega)$. For simplicity of notations, hereafter we denote by $\|\cdot\|_s$ the Lebesgue space $L^s(\Omega)$ norm and $\|\cdot\|$ denotes $L^2(\Omega)$ norm, we write equivalent norm $\|\Delta\cdot\|$ instead of $H_0^2(\Omega)$ norm $\|\cdot\|_{H_0^2(\Omega)}$. Moreover, C_i $(i = 1, 2, \cdots)$ denote various positive constants which depend on the known constants and may be difference at each appearance.

2 Preliminaries

To prove our main results, we make the following assumptions.

(A1) $g : R^+ \to R^+$ is a bounded C^1 function which satisfies

$$g(s) > 0,\ g'(s) \leq 0,\ l = 1 - \int_0^{+\infty} g(s)ds > 0,$$

and there exist positive constants η_1 and η_2 such that

$$-\eta_1 g(t) \leq g'(t) \leq -\eta_2 g(t).$$

(A2) Supposed that p and m satisfy the following conditions:

$$2 \leq p < +\infty,\ n \leq 4;\quad 2 < p \leq \frac{4}{n-4},\ n > 4. \tag{2.1}$$

$$2 \leq m < +\infty,\ n \leq 4;\quad 2 < m \leq \frac{8}{n-4},\ n > 4. \tag{2.2}$$

Now, we define the following functionals:

$$J(t) = \frac{1}{2}\left(1 - \int_0^t g(s)ds\right)\|\Delta u\|^2 + \frac{1}{2}(g \circ \Delta u)(t) - \frac{b}{p+2}\|u\|_{p+2}^{p+2}, \tag{2.3}$$

$$K(t) = \left(1 - \int_0^t g(s)ds\right)\|\Delta u\|^2 + (g \circ \Delta u)(t) - b\|u\|_{p+2}^{p+2}, \tag{2.4}$$

for $u \in H_0^2(\Omega)$, where

$$(g \circ \Delta u)(t) = \int_0^t g(t-s)\|\Delta u(s) - \Delta u(t)\|^2 ds.$$

Then we introduce the stable set W by

$$W = \{u \in H_0^2(\Omega): \quad K(t) > 0\} \cup \{0\}. \tag{2.5}$$

We denote the total energy related to the equation (1.1) by

$$\begin{aligned} E(t) \quad &= \frac{1}{2}||u_t||^2 + \frac{1}{2}\left(1 - \int_0^t g(s)ds\right)\|\Delta u\|^2 \\ &+\frac{1}{2}(g \circ \Delta u)(t) - \frac{b}{p+2}\|u\|_{p+2}^{p+2} = \frac{1}{2}||u_t||^2 + J(t) \end{aligned} \tag{2.6}$$

for $u \in H_0^2(\Omega),\ t \geq 0$ and $E(0) = \frac{1}{2}||u_1||^2 + J(0)$ is the initial total energy.

For latter applications, we list up some lemmas.

Lemma 2.1 Let s be a number with $2 \leq s < +\infty$ if $n \leq 4$ and $2 \leq s \leq \frac{2n}{n-4}$ if $n > 4$. Then there is a constant B_1 depending on Ω and s such that

$$\|u\|_s \leq B_1\|\Delta u\|,\ \forall u \in H_0^2(\Omega).$$

Lemma 2.2 Supposing that (A1) holds and that $u(t)$ is a solution to the problem (1.1), then $E(t)$ is a non-increasing function for $t > 0$ and

$$E'(t) = \frac{1}{2}(g' \circ \Delta u)(t) - \frac{1}{2}g(t)\|\Delta u\|^2 - a\|u_t\|_{m+2}^{m+2} \leq 0. \tag{2.7}$$

Proof Multiplying the equation in (1.1) by u_t, and integrating over $\Omega \times [0,t]$. Then we get from integrating by parts that

$$E(t) = E(0) + \int_0^t [\frac{1}{2}(g' \circ \Delta u)(s) - \frac{1}{2}g(s)\|\Delta u\|^2 - a\|u_t\|_{m+2}^{m+2}]ds \tag{2.8}$$

for $t \geq 0$. Being the primitive of an integrable function, $E(t)$ is absolutely continuous and equality (2.7) is satisfied.

We conclude this section by stating a local existence result of the problem (1.1), which can be established by combination of the arguments in [5, 7]. The readers are also referred to the references [8, 9].

Theorem 2.1(Local existence) Assuming that (A1) and (A2) hold, if $(u_0, u_1) \in H_0^2(\Omega) \times L^2(\Omega)$. Then there exists $T > 0$ such that the problem (1.1) has a unique local solution $u(t)$ which satisfies

$$u \in C([0,T];\ H_0^2(\Omega)), \quad u_t \in C([0,T];\ L^2(\Omega)) \cap L^{m+2}(\Omega \times [0,T)).$$

3 Global Solutions

To prove the global existence of solution to the problem (1.1), we need the following lemmas:

Lemma 3.1 Supposing that (A1) and (A2) hold, then

$$\frac{p}{2(p+2)}\left[\left(1-\int_0^t g(s)ds\right)\|\Delta u\|^2+(g\circ\Delta u)(t)\right]\leq J(t), \tag{3.1}$$

for $u\in W$.

Proof By the definition of $K(t)$ and $J(t)$, we have the following identity

$$(p+2)J(t)=K(t)+\frac{p}{2}\left[\left(1-\int_0^t g(s)ds\right)\|\Delta u\|^2+(g\circ\Delta u)(t)\right]. \tag{3.2}$$

Since $u\in W$, so we get $K(t)\geq 0$. Therefore, we obtain from (3.2) that (3.1) is valid.

Lemma 3.2 Let (A1) and (A2) hold, if $u_0\in W$ and $u_1\in L^2(\Omega)$ such that

$$\theta=\frac{bB_1^{p+2}}{l}\left[\frac{2(p+2)}{pl}E(0)\right]^{\frac{p}{2}}<1, \tag{3.3}$$

then $u(t)\in W$, for each $t\in[0,T)$.

Proof Since $u_0\in W$, so $K(0)>0$. Then it follows from the continuity of $u(t)$ that

$$K(t)\geq 0, \tag{3.4}$$

for some interval near $t=0$. Let $t_{\max}>0$ be a maximal time (possibly $t_{\max}=T$) when (3.4) holds on $[0,t_{\max})$.

We have from (2.6) and (3.1) that

$$\frac{p}{2(p+2)}\left[\left(1-\int_0^t g(s)ds\right)\|\Delta u\|^2+(g\circ\Delta u)(t)\right]\leq E(t), \tag{3.5}$$

We get from (A1) and Lemma 2.2 that

$$l\|\Delta u\|^2\leq\left(1-\int_0^t g(s)ds\right)\|\Delta u\|^2\leq\frac{2(p+2)}{p}E(0), \tag{3.6}$$

for $\forall t\in[0,t_{max})$.

By exploiting (A1), Lemma 2.1, (3.3) and (3.6), we easily arrive at

$$\begin{aligned}b\|u(t)\|_{p+2}^{p+2} &\leq bB_1^{p+2}\|\Delta u(t)\|^{p+2}\leq\frac{bB_1^{p+2}}{l}\|\Delta u(t)\|^p\cdot(l\|\Delta u(t)\|^2)\\ &\leq\frac{bB_1^{p+2}}{l}\left[\frac{2(p+2)}{pl}E(0)\right]^{\frac{p}{2}}\cdot(l\|\Delta u(t)\|^2)\leq\theta l\|\Delta u(t)\|^2\\ &\leq\theta\left(1-\int_0^t g(s)ds\right)\|\Delta u\|^2<\left(1-\int_0^t g(s)ds\right)\|\Delta u\|^2,\end{aligned} \tag{3.7}$$

for all $t \in [0, t_{\max})$. Therefore,

$$K(t) = \left(1 - \int_0^t g(s)ds\right)\|\Delta u\|^2 + (g \circ \Delta u)(t) - b\|u\|_{p+2}^{p+2} > 0$$

on $t \in [0, t_{\max})$. By repeating this procedure, and using the fact that

$$\lim_{t \to t_{\max}} \frac{bB_1^{p+2}}{l}\left[\frac{2(p+2)}{pl}E(t)\right]^{\frac{p}{2}} \le \theta < 1,$$

$t_{\max}$ is extended to T. Thus, we conclude that $u(t) \in W$ on $[0, T)$.

Theorem 3.1 Assuming that (A1) and (A2) hold and that $u(t)$ is a local solution as that obtained in Theorem 2.1. If $u_0 \in W$ and $u_1 \in L^2(\Omega)$ satisfy (3.3), then the solution $u(t)$ is a global and bounded solution of the problem (1.1).

Proof It suffices to show that $\|\Delta u(t)\|^2 + \|u_t(t)\|^2$ is bounded independently of t.

Under the hypotheses in Theorem 3.1, we get from Lemma 3.2 that $u(t) \in W$ on $[0, T)$. So the formula (3.1) in Lemma 3.1 holds on $[0, T)$.

Therefore, we have from (3.1) that

$$\begin{aligned}
&\frac{1}{2}\|u_t\|^2 + \frac{p}{2(p+2)}\left[l\|\Delta u\|^2 + (g \circ \Delta u)(t)\right] \\
&\le \frac{1}{2}\|u_t\|^2 + \frac{p}{2(p+2)}\left[\left(1 - \int_0^t g(s)ds\right)\|\Delta u\|^2 + (g \circ \Delta u)(t)\right] \\
&\le \frac{1}{2}\|u_t\|^2 + J(t) = E(t) \le E(0).
\end{aligned} \tag{3.8}$$

Hence, we get

$$\|u_t(t)\|^2 + \|\Delta u(t)\|^2 \le \max\left(2, \frac{2(p+2)}{pl}\right)E(0) < +\infty.$$

The above inequality and the continuation principle lead to the global existence of the solution, that is, $T = +\infty$. Thus, the solution $u(x, t)$ is a global solution of the problem (1.1).

4 Energy Decay of Global Solution

The following lemmas play an important role in studying the energy decay estimate of global solutions for the problem (1.1).

Lemma 4.1[10] Supposing that $\varphi(t)$ is a non-increasing and nonnegative function on $[0, T]$, $T > 1$, such that

$$\varphi(t)^{1+r} \le \omega_0[\varphi(t) - \varphi(t+1)], \ \textit{on} \ [0, T],$$

where ω_0 is a positive constant and r is a nonnegative constant. Then $\varphi(t)$ has the following decay properties

(i) if $r > 0$, then

$$\varphi(t) \leq \left(\varphi(0)^{-r} + \omega_0^{-1} r[t-1]^+\right)^{-\frac{1}{r}} \ \ on\ [0,T],$$

where $[t-1]^+ = \max\{t-1,\ 0\}$.

(ii) if $r = 0$, then

$$\varphi(t) \leq \varphi(0) e^{-\vartheta[t-1]^+} \ \ on\ [0,T],$$

where $\vartheta = \ln \frac{\omega_0}{\omega_0 - 1},\ \omega_0 > 1$.

Lemma 4.2 Let u satisfy the assumptions of Lemma 3.2. Then there exists $0 < \theta_1 < 1$ such that

$$\left(1 - \int_0^t g(s)ds\right)\|\Delta u(t)\|^2 \leq \frac{1}{\theta_1} K(t),\ t \in [0,T], \tag{4.1}$$

where $\theta_1 = 1 - \theta$.

Proof From (3.7), we get that

$$b\|u(t)\|_{p+2}^{p+2} \leq \theta\left(1 - \int_0^t g(s)ds\right)\|\Delta u(t)\|^2,\ t \in [0,T].$$

Let $\theta = 1 - \theta_1$, then

$$b\|u(t)\|_{p+2}^{p+2} \leq (1-\theta_1)\left(1 - \int_0^t g(s)ds\right)\|\Delta u(t)\|^2 + (g \circ \Delta u)(t),\ t \in [0,T]. \tag{4.2}$$

We have from (2.4) and (4.2) that (4.1) is valid.

Theorem 4.1 Under the assumptions of Theorem 3.1, if $u_0 \in W$ and $u_1 \in L^2(\Omega)$ satisfy (3.3), then the global solution $u \in W$ of the problem (1.1) satisfies the following decay properties:

(i) If $m = 0$, then $E(t) \leq E(0)e^{-\vartheta[t-1]^+}$.

(ii) If $m > 0$, then

$$E(t) \leq \left(E(0)^{-\frac{m}{2}} + \hbar[t-1]^+\right)^{-\frac{2}{m}}.$$

where ϑ and $\hbar$ are positive constants which will be determined later.

Proof Multiplying the equation in (1.1) by u_t and integrating over $\Omega \times [t, t+1]$, we get

$$\begin{aligned} & a\int_t^{t+1} \|u_t(s)\|_{m+2}^{m+2} ds - \frac{1}{2}\int_t^{t+1} (g' \circ \Delta u)(s)ds \\ & + \frac{1}{2}\int_t^{t+1} g(s)\|\Delta u(s)\|^2 ds = E(t) - E(t+1). \end{aligned} \tag{4.3}$$

Thus, there exist $t_1 \in \left[t, t+\frac{1}{4}\right],\ t_2 \in \left[t+\frac{3}{4}, t+1\right]$ such that

$$\begin{aligned} & 4a\|u_t(t_i)\|_{m+2}^{m+2} - 2(g' \circ \Delta u)(t_i) \\ & + 2g(t_i)\|\Delta u(t_i)\|^2 = E(t) - E(t+1),\ t = 1, 2. \end{aligned} \tag{4.4}$$

On the other hand, we multiply the equation in (1.1) by u and integrate over $\Omega\times[t_1,t_2]$. We obtain

$$\begin{aligned}\int_{t_1}^{t_2}K(t)ds \quad &= \int_{t_1}^{t_2}\|u_t(s)\|^2ds+(u_t(t_1),u(t_1))-(u_t(t_2),u(t_2))\\ &-a\int_{t_1}^{t_2}\int_\Omega|u_t|^m u_t u dxds+\int_{t_1}^{t_2}(g\circ\Delta u)(s)ds\\ &+\int_{t_1}^{t_2}\int_\Omega\int_0^t g(t-s)\Delta u(t)[\Delta u(s)-\Delta u(t)]dsdxdt.\end{aligned}\tag{4.5}$$

From (4.3) and Hölder inequality, we have

$$\int_{t_1}^{t_2}\|u_t\|^2ds\le C_3\bigg(\int_{t_1}^{t_2}\|u_t\|_{m+2}^{m+2}ds\bigg)^{\frac{2}{m+2}}\le C_3[E(t)-E(t+1)]^{\frac{2}{m+2}}.\tag{4.6}$$

We get from (3.8), (4.4), Hölder inequality and Young inequality that

$$\begin{aligned}&|(u_t(t_i),u(t_i))|\le\|u_t(t_i)\|\|u(t_i)\|\\ &\le C_4(E(t)-E(t+1))^{\frac{1}{m+2}}\sup_{t\le s\le t+1}E(s)^{\frac12},\ i=1,2.\end{aligned}\tag{4.7}$$

From Hölder inequality and Lemma 2.1, (3.8) and (4.3), we get

$$\left|\int_{t_1}^{t_2}\int_\Omega|u_t|^m u_t u dxds\right|\le C_5(E(t)-E(t+1))^{\frac{m+1}{m+2}}\sup_{t\le s\le t+1}E(s)^{\frac12}.\tag{4.8}$$

By using Young inequality, we have

$$\begin{aligned}&\int_{t_1}^{t_2}\int_\Omega\int_0^t g(t-s)\Delta u(t)[\Delta u(s)-\Delta u(t)]dsdxdt\\ &\le\sigma\int_{t_1}^{t_2}\int_0^t g(t-s)\|\Delta u(t)\|^2dsdt+\frac{1}{4\sigma}\int_{t_1}^{t_2}(g\circ\Delta u)(s)ds,\end{aligned}\tag{4.9}$$

where σ is some positive constant to be chosen later.

Therefore, we get from (4.5)-(4.9) that

$$\begin{aligned}&\int_{t_1}^{t_2}K(t)ds\\ &\le C_6\bigg[(E(t)-E(t+1))^{\frac{2}{m+2}}+(E(t)-E(t+1))^{\frac{m+1}{m+2}}\sup_{t\le s\le t+1}E(s)^{\frac12}\\ &+(E(t)-E(t+1))^{\frac{1}{m+2}}\sup_{t\le s\le t+1}E(s)^{\frac12}\bigg]\\ &+\sigma\int_{t_1}^{t_2}\int_0^t g(t-s)\|\Delta u(t)\|^2dsdt+\left(\frac{1}{4\sigma}+1\right)\int_{t_1}^{t_2}(g\circ\Delta u)(s)ds.\end{aligned}\tag{4.10}$$

On the other hand, we obtain from (A1) and (4.3) that

$$\int_{t_1}^{t_2} (g \circ \Delta u)(s)ds \leq -\frac{1}{\eta_2} \int_{t_1}^{t_2} (g' \circ \Delta u)(s)ds \leq \frac{2}{\eta_2}(E(t) - E(t+1)). \tag{4.11}$$

By (A1) and Lemma 4.2, we have

$$\int_{t_1}^{t_2} \int_0^t g(t-s)\|\Delta u(t)\|^2 dsdt \leq \frac{g(0)}{l\theta_1\eta_2} \int_{t_1}^{t_2} K(t)dt. \tag{4.12}$$

Choosing σ such that $\frac{\sigma g(0)}{l\theta_1\eta_2} = \frac{1}{2}$, then we get from (4.10), (4.11) and (4.12) that

$$\begin{aligned} &\int_{t_1}^{t_2} K(t)ds \\ &\leq C_7 \Big[(E(t) - E(t+1))^{\frac{2}{m+2}} + (E(t) - E(t+1))^{\frac{1}{m+2}} \sup_{t \leq s \leq t+1} E(s)^{\frac{1}{2}} \\ &+ (E(t) - E(t+1))^{\frac{m+1}{m+2}} \sup_{t \leq s \leq t+1} E(s)^{\frac{1}{2}} + (E(t) - E(t+1)) \Big]. \end{aligned} \tag{4.13}$$

It follows from (2.3), (2.4) and Lemma 4.2 that

$$J(t) \leq \frac{p}{2(p+2)}(g \circ \Delta u)(t) + \frac{p+2\theta_1}{2(p+2)\theta_1} K(t). \tag{4.14}$$

We have from (2.6) and (4.14) that

$$E(t) \leq \frac{1}{2}\|u_t\|^2 + \frac{p}{2(p+2)}(g \circ \Delta u)(t) + \frac{p+2\theta_1}{2(p+2)\theta_1} K(t). \tag{4.15}$$

By integrating (4.15) over $[t_1, t_2]$, we obtain from (4.6), (4.11) and (4.13) that

$$\begin{aligned} &\int_{t_1}^{t_2} E(t)ds \\ &\leq 2C_8[(E(t) - E(t+1))^{\frac{2}{m+2}} + (E(t) - E(t+1))] \\ &+ C_8[(E(t) - E(t+1))^{\frac{m+1}{m+2}} + (E(t) - E(t+1))^{\frac{1}{m+2}}] \sup_{t \leq s \leq t+1} E(s)^{\frac{1}{2}}. \end{aligned} \tag{4.16}$$

Integrating both sides of (2.7) over $[t, t_2]$, we obtain from (4.3) and $E(t_2) \leq 2\int_{t_1}^{t_2} E(s)ds$ that

$$E(t) \leq 2\int_{t_1}^{t_2} E(s)ds + (E(t) - E(t+1)). \tag{4.17}$$

We have from (4.16) and (4.17) that

$$\begin{aligned} E(t) \quad &\leq 2C_8[(E(t) - E(t+1))^{\frac{m+1}{m+2}} + (E(t) - E(t+1))^{\frac{1}{m+2}}]E(t)^{\frac{1}{2}} \\ &+ C_9[(E(t) - E(t+1))^{\frac{2}{m+2}} + (E(t) - E(t+1))]. \end{aligned} \tag{4.18}$$

Therefore, we obtain from Young inequality that

$$\begin{aligned} E(t) &\leq C_{10}[(E(t)-E(t+1)) \\ &+(E(t)-E(t+1))^{\frac{2(m+1)}{m+2}}+2(E(t)-E(t+1))^{\frac{2}{m+2}}]. \end{aligned} \tag{4.19}$$

When $m = 0$, we have

$$E(t) \leq 4C_{10}[(E(t)-E(t+1))]. \tag{4.20}$$

Applying Lemma 4.1 to (4.20), we get

$$E(t) \leq E(0)e^{-\vartheta[t-1]^+},$$

where $\vartheta = \ln \frac{4C_{10}}{4C_{10}-1}$.

When $m > 0$, we obtain from (4.19) that

$$E(t) \leq C_{11}[E(t)-E(t+1)]^{\frac{2}{m+2}}. \tag{4.21}$$

We have from (4.21) that

$$E(t)^{\frac{m+2}{2}} \leq C_{12}[E(t)-E(t+1)]. \tag{4.22}$$

Consequently, we obtain from (4.22) and Lemma 4.1 that

$$E(t) \leq \left(E(0)^{-\frac{m}{2}} + \hbar[t-1]^+\right)^{-\frac{2}{m}}, \tag{4.23}$$

where $\hbar = \frac{m}{2C_{12}}$.

Thus, we complete the proof of Theorem 4.1.

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